BLOCK V:
STATISTICAL DECISION THEORY

Unit-1
Decision Theory Concepts

Unit Structure

1.1 Introduction

In this unit, you will learn about decision theory also known as decision analysis. The basic function of any business manager is decision making for various purposes. The quality of decision making depends on the quantity and quality of information available to him or utilised by him. Decision theory is a systematic approach for such a function. Elements of the decision process are alternative courses of action, states of nature, pay-off and pay-off table.

1.2 Objectives

After going through this unit, you will be able to-

- Learn about the elements of a decision process,
- Learn about payoff matrix and loss table.

1.3 Decision Theory Concepts

Statistical decision theory provides an analytical and systematic approach to the study of decision making wherein data concerning the occurrence of different outcomes (consequence) may be evaluated to enable the decision maker to identify a suitable decision alternative (or course of action). Irrespective of the type of decision model, there are certain essential characteristics which are common to all as listed below:

1. Acts- A decision maker has to first determine the alternative course of action so called, acts of strategies from which he has to choose one considered to be the best.
2. States of nature – The circumstances, which affect the outcome of a decision problem but are beyond the control of the decision maker are known as states of nature or events.
3. Payoff – A numerical value resulting from each possible combination of alternatives and states of nature is called payoff. The payoff values are always conditional values because of an unknown states of nature.

Check Your Progress

1. What is a pay off table ?
2. What is opportunity loss?

A tabular arrangement of these conditional outcomes (payoff) values is known as payoff matrix as shown in Table 8.1

Table 1.1 General form of Payoff Matrix

	Courses of action (Alternatives)				
States of Nature	S_1	S_2	...	...	S_n
N_1	P_{11}	P_{12}			P_{1n}
N_2	P_{21}	P_{22}			P_{2n}
...					
...					
N_n	P_{m1}	P_{m2}			P_{mn}

4. Opportunity Loss table – The opportunity loss is defined to be the difference between the highest possible profit for a state of nature and the actual profit obtained for the particular decision taken i.e. an opportunity loss is the loss incurred due to the failure of not adopting the best possible course of action or strategy. The pay off table which represents the cost or loss incurred because of failure

to take the best possible action is called the opportunity loss table. If for a given state of nature N_i , the n^{th} pay offs corresponding to the n courses of action are given by $p_{i1},p_{i2},...,p_{in}$ and if M stands for the maximum of these quantities the respective opportunity losses asre computed as shown in the table.

Table 1.2 Regret (Opportunity loss table)

	Courses of action (Conditional Opportunity Loss)				
States of Nature	S_1	S_2	...	...	S_n
N_1	M_1-P_{11}	M_1-P_{12}			M_1-P_{1n}
N_2	M_2-P_{21}	M_2-P_{22}			M_2-P_{2n}
...					
...					
N_n	M_m-P_{m1}	M_m-P_{m2}			M_m-P_{mn}

Stop to Consider

Decision theory is a systematic approach for decision making depending on the quality and quantity of information available to him or utilised by him.

Various elements of a decision-making process include decision alternatives, state of nature, payoff, pay-off table and opportunity loss table.

1.4 Summing Up

A decision may be defined as the process of choosing an alternative course of action , given that atleast two alternatives exist.

State of nature refers to a future event not under the control of the decision maker.

Payoff is the benefit that accrues from a given combination of a decision alternative and a state of nature.

Opportunity loss refers to the profit that could have been earned if stock had been sufficient to supply a unit that was demanded.

Alternative courses of action

Conditional profit values are the profit that would result from a given combination of decision alternatives and state of nature.

1.5 Key Terms

Decision: It is a conclusion reached after due consideration.

Alternative courses of action: It refers to a choice of two or more possibilities of things, propositions, the selection of which preclude any other possibility:

1.6 Model Questions

A. Multiple Choice Questions

1. Elements of the decision process include

   (a) pay-off (b) pay-off table

   (c) regret table (d) all of the above

2. Benefit that accrues from a given combination of a decision alternative and a state of nature.

   (a) pay-off (b) loss

   (c) alternatives (d) all of the above

B. Fill in the blanks

1. A _____may be defined as the process of choosing an alternative course of action .

2. _____refers to a choice of two or more possibilities of things.

C. State whether True or False

1. A payoff table cannot include the probability value for each event .

2. Acts are referred to as the strategies available to the decision maker

D. Match Column A with Column B

	Column A		Column B
1	It is an element of decision process.	A	Strategies available to the decision maker
2	Acts are	B	Opportunity Loss Table

E. Short Answer Questions

1. Define a pay off table.
2. Define opportunity loss table.
3. What are the elements of decision making process?

F. Long –Answer Questions

1. Discuss the various elements of the decision making process.
2. Calculate the loss table from the following pay-off table.

Action	Events			
	E_1	E_2	E_3	E_4
A_1	50	300	-150	50
A_2	400	0	100	0
A_3	-50	200	0	100
A_4	0	300	300	0

3. What is a pay-off table?
4. A baker produces a certain type of special pastry at an average cost of Rs 3 and sells it a price of Rs 5. This pastry is produced over the weekend and is sold during the following week, such pastry being produced but not sold during a week's time are totally spoiled and have to be thrown away. According to past experience the weekly demand for these pastries is never less than 78 or greater than 80 . Formulate action space, pay-off table and loss table

1.7 References and Suggested Readings

Hazarika, P.L. Essential Statistics for Economics and Business Statistics.New Delhi. Akansha Publishing House,2012

Sharma.J.K Business Statistics .Pearson Education , New Delhi,2007

Gupta,S.C Fundamental of Statistics , New Delhi : S.Chand and sons ,2005

Gupta, S. P .Statistical Methods .New Delhi . S.Chand and Sons,2005

Hooda, R.P . Statistics for Business and Economics. New Delhi: Macmillan India Ltd 2002

Sharma Anand. Statistics for Management, Himalaya Publishing House, Geetanjali Press Pvt . Ltd, Nagpur.

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Unit-II

Steps in Decision Making Process

Unit Structure:

2.1 Introduction

In this unit, you will learn about decision theory also known as decision analysis. A decision may be defined as the process of choosing an alternative course of action given that at least two alternatives exist. A decision is taken under three situations, viz (a) certainty (b) uncertainty and (c) risk. It is trivial to take a decision under certainty because a decision-maker knows what will be the result when a particular decision is taken. A decision under uncertainty means the choice of action out of the many courses of action at hand when the outcome of any action is unknown . In decision making, under risk, more than one state of nature and decision-maker has sufficient information to assign probabilities to each of these states.

2.2 Objectives

After going through this unit, you will be able to

- Learn about the steps in the decision making process
- Explain about the various types of decision –making environment

2.3 Steps in the Decision Making Process

The decision making process involves the following steps :

1. Identifying and defining the problem
2. Listing of all possible future events, called states of nature, which can occur in the context of decision problem.
3. Identification of all the course of action which are available to the decision maker.
4. Construct a pay off table for each possible combination of alternatives course of action and state of nature.
5. Choose the criterion that results in the largest pay –off.

2.4 Decision Making Environment

Decisions are made under three types of environments:

1. Decision making under condition of certainty: In this case the decision maker has complete knowledge (perfect information) of the consequence of every decision choice (alternative) with certainity. Obviously, he will select an alternative that yields the largest payoff.
2. Decision making under uncertainity : In this case the decision maker is unable to specify the probabilities with which the various states of nature (future) will occur.
3. Decision making Under conditions of Risk –As in case of decision making under conditions of uncertainity here also more than one states of nature exist. But here the decision maker has sufficient information to allow him to assign probabilities to the various states of nature.

   Under conditions of uncertainity , the decision maker has knowledge about states of nature that happens but lacks the knowledge about the probabilities of their occurrence.

   Under conditions of uncertainity , a few decision criterions are available which could be of help to the decision maker.

   (1) Maximax Criteron or Criterion of Optimism: This criterion provides the decision maker with optimistic criterion . The working method is summarised as follows:

   - Locate the maximin payoff value corresponding to each alternative

- Select an alternative with maximin payoff

(2) Maximin Criterion or Criterion of Pessimism: This criterion provides the decision maker with pessimistic criterion. The working method is summarised as follows:

- Locate the minimum payoff value corresponding to each alternative
- Select an alternative with maximum pay-off value

(3) Minimax Criterion or Minimum Regret Criterion : This criterion is also known as opportunity loss decision criterion or minimax regret criterion . The working method is summarised as follows:

Determine the amount of regret corresponding to each alternative for each state of nature.

- The regret for j^{th} event corresoponding to the i^{th} alternative is given by i^{th} regret =(maximum payoff –ith payoff) for j^{th} event
- Determine the maximum regret amount for each alternative
- Choose the alternative which corresponds to the minimum of the maximum regrets.

(4) Hurwicz Criterion or Criterion of Realism : Also called weighted average criterion , it is a compromise between the maximax (optimistic) and minimax (pessimistic) decision criterion . This concept allows the decision maker to take into account both maximum and minimum for each alternative and assign them weights according to his degree of optimism (0r pessimism). The working method is summarised as follows:

- Choose an appropriate degree of optimism , α so that $(1-\alpha)$ represents degree of pessimism
- Determine the maximum as well as minimum of each alternative and obtain

  $P = \alpha$. Maximum $+(1-\alpha)$ Minimum for each alternative
- Choose the alternative that yields the maximum value of P

(5) Laplace Criterions or Criterion of Rationality – Also known as equal probabilities criterions or criterion of rationality . Since

the probability of states of nature are not known , it is assumed that all states of nature will occur with equal probability . The working method is summarised as follows:

- Determine the expected value for each alternative ; if n denotes the number of events and P_i's denote the payoffs, then expected value is given by

$$\frac{1}{n}[P_1 + P_2 + ... + P_n]$$

Stop to Consider

Payoff – A numerical value resulting from each possible combination of alternatives and states of nature is called payoff.

Decisions are made under three types of environments :-

1. Decision making under condition of certainity

2. Decision making under condition of uncertainity

3. Decision making under condition of risk

Choose the alternative that yields the maximum value of P.

Types of biscuits	Profits on estimated level of sales (in lakhs) for quantities		
	5,000	10,000	20,000
Marie Gold(M)	15	25	45
Good Day(G)	20	55	65
Oreo(O)	25	40	70

Example 1: A company has to choose one of the three types of biscuits , Marie Gold , Good day , Oreo. Sales expected during next year are highly uncertain .marketing Department estimates the profits considering manufacturing cost , promotional efforts and distribution set up as given in the table below.

Solve using decision making under uncertainity.

Solution : Maximin criterion : In using the maximin criterion , the decision maker adopts a pessimistic approach and tries to maximise his security in the face of a highly uncertain situation. For worst situation , the payoffs are 15, 20 and 25 for M,G and O respectively. Even at the pessimistic level , the manager tries to make the best of the situation reaching the decision as

Oreo , pay off being the best amongst the worst. This maximises the minimum pay-off. Hence, the company will launch Oreo(O).

MinimaxCriterion : Here, the maximum pay-offs are 45,65 and 70 in three cases. In order to gain atleast the minimum of these maximams , minimum pay off is 45 for Marie gold (M). Hence the strategy will be to launch Marie Gold(M).

MaximaxCriterion : In this case , the decision maker become totally optimistic and chooses the strategy that makes the best of the best . The largest payoff for each type are 45, 65 and 70. The maximum payoff being 70, the company choose to launch Oreo(O).

Laplace Criterion : when decision maker has no definite information about the probability of occurrence of various states of nature , he makes simple assumption that each is equally likely. Therefore , the probability of each to occur is 1/3.

Expected pay-offs are :

$$E(M) = \frac{1}{3}\times 15 + \frac{1}{3}\times 25 + \frac{1}{3}\times 45 = 28.33$$

$$E(G) = \frac{1}{3}\times 20 + \frac{1}{3}\times 55 + \frac{1}{3}\times 65 = 46.67$$

$$E(O) = \frac{1}{3}\times 25 + \frac{1}{3}\times 40 + \frac{1}{3}\times 70 = 45$$

The strategy for Good Day expects the maximax pay-off. Hence, the decision would be to launch Good Day biscuit.

Hurwicz Criterion : Maximin and Maximax are two extremes on the scale of optimism . It would be pragmatic to assume that a business manager's attitude would fall somewhere in between rather than at either extremes. Hurwicz ,therefore, propounds a combination of the two criteria in what is known as Hurwicz Alpha criterion.

In this case , the decision maker's degree of optimism is represented by α, the coefficient of optimism , varying between 0 and 1 ; $\alpha=0$ denoting total pessimism and $\alpha=1$, total optimism .

A decision index D_i is defined by

$D_i=\alpha M_i+(1-\alpha)m_i$ where

M_i=Max. pay-offs from any of the outcomes resulting from the i^{th} strategy

m_i= min payoff from any of the outcomes resulting from the i^{th} strategy

For each strategy , the value of decision index is found and strategy with the highest values of outcome is chosen .

Let us assume α=0.6 in this example

$D_i(M)=(0.6x45)+(1-0.6)x15=33$

$D_i(G)=(0.6x65)+(1-0.6)x20=47$

$D_i(O)=(0.6x70)+(1-0.6)x25=52$

The strategy chosen , therefore , would be to launch Good day (G) producing the best outcome i.e. 52

Regret Criterion : Loss of opportunity is a common phenomenon in the business world. The Regret Criterion , the dissatisfaction associated with not having got the best that would have been possible if the state of nature of occurrence were known in advance.

A measure of regret of an outcome is the opportunity cost computed as the difference in pay-off of the outcome and the largest pay-off which could have been obtained under the corresponding state of nature . This table so obtained is called Opportunity Loss Table (OL table)

Revised pay-off (subtracting pay-off from highest of that event)i.e. regret pay-off

	5000	10,000	20,000	Max. regret
Marie Gold(M)	25-15	55-25	70-45	30
Good Day(G)	25-20	55-55	70-65	5
Oreo(O)	25-25	55-40	70-70	15

Minimum of maximum regret is 5 corresponding to Good day biscuits .Hence it is decided to launch Good Day(G).

The choice of the decision maker is the reason for inconsistency in the above results . The personality of the decision maker plays an important role in these decisions.

Stop to Consider

Criterions for decision making under uncertainity-

Maximax Maximin Hurwicz

Minimax Laplace Regret

Example 2: Consider a bottling company that is thinking of various alternatives to increase its production to meet the increasing market demand. Use the various criterions of decision making under uncertainity to arrive at a solution.

Alternatives	State of nature (Product Demand)			
	High	Moderate	Low	Nil
Expand	50,000	25,000	-25,000	-45,000
Construct	70,000	30,000	-40,000	-80,000
Subcontract	30,000	15,000	-1,000	-10,000

Solution : (1) Maximin Criterion

Alternatives	State of nature (Product Demand)				Maximum of Rows
	High	Moderate	Low	Nil	
Expand	50,000	25,000	-25,000	-45,000	50,000
Construct	70,000	30,000	-40,000	-80,000	70,000
Subcontract	30,000	15,000	-1,000	-10,000	30,000

Thus the maximax payoff is rs 70,000 corresponding to the alternative " construct"

(ii) Maximin Criterion

Alternatives	State of nature (Product Demand)				Minimum of Rows
	High	Moderate	Low	Nil	
Expand	50,000	25,000	-25,000	-45,000	-45,000
Construct	70,000	30,000	-40,000	-80,000	-80,000
Subcontract	30,000	15,000	-1,000	-10,000	-10,000

Thus the maximin payoff is Rs -10,000 corresponding to the alternative 'subcontract'

(iii)) Minimax Criterion or Minimum Regret Criterion

Alternatives	State of nature (Product Demand)			
	High	Moderate	Low	Nil
Expand	50,000	25,000	-25,000	-45,000
Construct	70,000	30,000	-40,000	-80,000
Subcontract	30,000	15,000	-1,000	-10,000

Calculation of Regret

Alternatives	State of nature (Product Demand)				Maximum of Rows
	High	Moderate	Low	Nil	
Expand	20,000	5000	24,000	35,000	-45,000
Construct	0	0	39000	70000	-80,000
Subcontract	40000	15000	0	0	-10,000

This table shows that the company will minimize its regret to Rs 35,000 by selecting alternative 'expansion'

(iv) Hurwicz Criterion

Let $\alpha=0.8$

Alternatives	State of nature (Product Demand)				Max of rows	Min of rows	$H=\alpha$ max$+(1-\alpha)$ min
	High	Moderate	Low	Nil			
Expand	20,000	5000	24,000	35,000	50,000	-45,000	31000
Construct	0	0	39000	70000	70,000	-80,000	40000
Subcontract	40000	15000	0	0	30,000	-10,000	22000

Thus according to the Hurwicz Criterions , the company will choose alternative ' construct'

(v) Laplace Criterion

Alternatives	State of nature (Product Demand)				Expected Pay Off
	High	Moderate	Low	Nil	
Expand	20,000	5000	24,000	35,000	1250
Construct	0	0	39000	70000	-5000
Subcontract	40000	15000	0	0	8500

Note : (EP_1) =1/4[50,000+25,000+(-25000)+(-45000)]=1250 etc.

Decision making Under Risk :

Here more than one state of nature exists and the decision maker has sufficient information to assign probabilities to each of these states. These probabilities could be obtained from the past records or simply the subjective judgement of the decision maker. Under conditions of risk , a number of decision criterions are available which could be of help to the decision maker.

(1) Expected Value Criterions – The expected monetary value for a given course of action is the weighted sum of possible payoffs for each alternative . It is obtained by summing the payoffs for each course of action multiplied by the probabilities associated with state of nature. It consist of the following steps:

- Construct a payoff table listing the alternative decisions and the various state of nature . Enter the conditional profit for each decision event combination along with the associated probabilities.
- Calculate the EMV for each decision alternative by multiplying the conditional profits by assigned probabilities and adding the resulting conditional values.
- Select the alternative that yields the highest EMV.

(2) Expected Opportunity Loss (EOL) Criterion : In this approach , first construct a conditional profit table for each decision –event combination along with the associated probabilities . for each event, compute the conditional opportunity loss by subtracting the corresponding payoff from the maximum payoff for that event .

Calculate the expected opportunity loss for each alternative by multiplying the conditional opportunity losses by the assigned probabilities and summing up their product . The alternative that yields the lowest EOL is selected.

(3) Expected Value of Perfect Information (EVPI) Perfect information means complete and accurate information about the future demand and that remove all the uncertainity for future.

EVPI represents the maximum amount of money the decision maker has to pay to get this additional information about the occurrence of various state of nature before a decision has to be made . The procedure to calculate the expected value of perfect information is as follows:

- Construct conditional profit table with perfect information
- Construct expected profit table with perfect information
- Determine EVPI from the following relation

$$EVPI = EPPI - \max EMV$$

Check Your Progress

1. What are the elements of decision making process?

2. Name some criterion that helps in decision making under uncertainity.

3. Name some decision criterion under the condition of Risk.

Example 3: A newspaper boy has the following probabilities of selling a magazine

No of copies sold	Probability
10	0.10
11	0.15
12	0.20
13	0.25
14	0.30

Cost of the copy is 30 paisa and sale price is 50 paise. He cannot return the unsold copies. How many should he order?

Solution

(1) EMV Criterion

CP =30 p

SP = 50 p

Profit = SP-CP=20 p

We construct the Conditional Profit table

Conditional Profit = Profit *S.P =20 S.P when D≥S

= 50 D-30 S, when D<S

Possible Demand	Probability	Possible Stock				
		10	11	12	13	14
10	0.10	200	170	140	110	80
11	0.15	200	220	190	160	130
12	0.20	200	220	240	210	180
13	0.25	200	220	240	260	230
14	0.30	200	220	240	260	280

The news boy must , therefore , order 12 copies to earn the highest possible average daily profit of 222.50 paise

(2) Expected Opportunity Loss

Using the conditional profit table , the conditional loss table is prepared by the following relation

Row maximum – other elements of row

Possible Demand	Probability	Possible Stock				
		10	11	12	13	14
10	0.10	200	170	140	110	80
11	0.15	200	220	190	160	130
12	0.20	200	220	240	210	180
13	0.25	200	220	240	260	230
14	0.30	200	220	240	260	280

Opportunity Loss table

Possible Demand	Probability	Possible Stock				
		10	11	12	13	14
10	0.10	0	3	6	9	12
11	0.15	3	0	4.5	9	13.5
12	0.20	8	4	0	6	12
13	0.25	15	10	5	0	7.5
14	0.30	24	18	12	6	0
EOL		50	35	27.5	30	45

The optimum stock action is the one which minimizes the expected opportunity loss. Therefore the newspaper boy should keep a stock of 12 copies where there is a minimum expected loss of 27.5 paise.

(iii) Expected profit table With Perfect Information

Possible Demand	Probability	Conditional profit Under Certainity	Expected profit With Perfect Information
10	0.10	200	20
11	0.15	220	33
12	0.20	240	48
13	0.25	260	65
14	0.30	280	84

The expected value of perfect information is given by

EVPI= EPPI-max EMV

= 250-222.5

= 27.5 paise

Thus this is the maximum amount which the newspaper boy is willing to pay per day for perfect information .

Example 4: A manufacturer is faced with a problem of fast change of technology and hence , fast change in the product line . At this point of time , the research and development wing of the organization has suggested an improved new product line with easy acceptance . It will cost the manufacturer Rs 60,000 for the pilot testing and development testing before establishing the product in the market . The organisation has 100 customers and each customer , might purchase , at the most ,one unit of the product , , due to its cost and newness . The selling price suggested is Rs 6,000 for each unit and selling estimate is Rs 2,000 for each unit. The probability distribution for proportion of customers buying the product is estimated as follows:

Proportion of Customers	Probability
0.04	0.1
0.08	0.1
0.12	0.2
0.16	0.4
0.20	0.2

Work out the expected opportunity losses and suggest whether the manufacturer should develop the product or not .

Solution : Let p be the proportion of customers , who purchase the new product . The conditional profit would be governed by the relationship as

(6,000-2,000)x100p-(60,000)=Rs (40,000p-60,000)

The conditional profit and opportunity loss are given as under :

Nature of state (proportion of customers)	Probability	Conditional Profit		Opportunity Loss (Rs.)	
		A_1(develop the product)	A_2(donot develop the product)	A_1	A_2
0.04	0.1	-44,000	0	44,000	0
0.08	0.1	-28,000	0	28,000	0
0.12	0.2	-12,000	0	12,000	0
0.16	0.4	4,000	0	0	4,000
0.20	0.2	20,000	0	0	20,000

Hence, EOL (A_1)=44,000x0.1+28,000x0.1+12,000x0.2+ 0x0.04+0x0.02

= Rs 9,600

To seek minimisation of opportunity loss, the manufacturer should not develop the product.

2.5 Summing Up

Maximax criterion- For uncertain conditions, a decision made based on best of the best alternatives

Maximin criterion-A decision under uncertainity using the best opportunity for the most pessimistic outcomes.

Laplace criterion –Under uncertain conditions, when we use equal probability for all opportunities it is , called Laplace criterion of decision making.

Expected Opportunity Loss- It is the minimum value expected outcome of the situation from all the available alternatives. It is the product of the conditional pay-off with the corresponding value of the probability.

EVPI- In probabilistic situations, the improvement in the expected value of the outcome due to better available information.

2.6 Key Terms

Decision: It is a conclusion reached after due consideration.

Alternative courses of action: It refers to a choice of two or more possibilities of things, propositions, the selection of which preclude any other possibility:

Decision under uncertainity: Here more than one state of nature exists but the decision maker lacks the knowledge about the probabilities of their occurrence.

Decision under risk: Here more than one state of nature exists and the decision-maker has sufficient information to assign probabilities to each of these states.

2.7 Model Questions

A. Multiple Choice Questions

1. A type of decision making environment is

   (a) certainty (b) uncertainty

   (c) risk (d) all of these

2. Elements of the decision process include

   (a) pay-off (b) pay-off table

   (c) regret table (d) all of the above

3. Which of the following needs the coefficient of optimism (α)

   (a) equally likely (b) maximin

   (c) realism (d) minimax

4. Which of the following is not used for decision making under uncertainty?

   (a) maximin (b) maximax

   (c) minimax (d) minimize expected loss

5. The value of the coefficient of optimism (α) is needed while using the criterion of

   (a) equally likely (b) maximum

   (c) realism (d) minimax

B. Fill in the blanks

1. Decision making under _____ refer to situations where more than one outcome can result from any single decision.
2. Coefficient of _____ is needed while using the criterion of realism .
3. _____has more than one state of nature exists and the decision-maker has sufficient information to assign probabilities to each of these states.

C. State whether True or False

1. Hurwicz criterion is also called the weighted average criterion .
2. Maximax criterion comes under decision making under risk.

D. Match the following

	Column A		Column B
1.	It is an element of decision process.	A.	Laplace Criterion
2.	It is helpful for decision maker	B.	Decision making under certainty
3.	It is based on what is known as the principle of insufficient reason.	C.	Opportunity loss table
4.	Whenever there exists only one outcome for a decision, we are dealing with this category	D.	Decision tree

E. Short Answer Questions

1. Define Hurwicz criterion.
2. Define EOL.
3. What is EVPI?
4. Give example of a decision taken under certainty.

F. Long –Answer Questions

1. Discuss the various elements of the decision making process.
2. Discuss the difference between decision making under certainty, uncertainty and risk .
3. Briefly explain 'expected value of perfect information' with examples.
4. A product of boats has estimated the following distribution of demand for a particular kind of boat:

No. demanded	0	1	2	3	4	5	6
Probability	0.14	0.27	0.27	0.18	0.09	0.04	0.01

   Each boat costs him Rs 7000 and he sells them for Rs 10,000 each . Boats left unsold at the end of the season must be disposed of for Rs 6000 each . How many boats should be in stock so as to maximise his expected profit ?

5. Calculate the loss table from the following pay-off table.

Action	Events			
	E_1	E_2	E_3	E_4
A_1	50	300	-150	50
A_2	400	0	100	0
A_3	-50	200	0	100
A_4	0	300	300	0

Suppose that the probabilities of the events in this table are

$P(E_1)=0.15$ $P(E_2)=0.45$ $P(E_3)=0.25$ $P(E_4)=0.15$

Calculate the expected pay-off and expected loss to each action.

6. The estimated sales of proposed types of perfumes are as under:

Types of perfumes	Estimated Sales		
	Rs 20,000	Rs 10,000	Rs 2,000
A	25	15	10
B	40	20	5
C	60	25	3

Help the manager to take effective decisions under minimax and Laplace method.

2.8 References and Suggested Readings

Hazarika, P.L. Essential Statistics for Economics and Business Statistics.New Delhi. Akansha Publishing House,2012

Sharma.J.K Business Statistics .Pearson Education , New Delhi,2007

Gupta,S.C Fundamental of Statistics , New Delhi : S.Chand and sons ,2005

Gupta, S. P .Statistical Methods .New Delhi . S.Chand and Sons ,2005

Hooda, R.P . Statistics for Business and Economics. New Delhi: Macmillan India Ltd 2002

Sharma Anand. Statistics for Management, Himalaya Publishing House, Geetanjali Press Pvt . Ltd, Nagpur

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Unit-3
Decision Tree

Unit Structure:

3.1 Introduction

In this unit you will learn about decision trees. Decision tree analysis involves the construction of a diagram showing all the possible courses of action, states of nature, and the probabilities associated with the state of nature. The decision diagram looks very much like the drawing of a tree , therefore also called a decision –tree. The basic advantage of decision tree approach is that it structures the decision process and helps decision making in an orderly, systematic and sequential manner.

3.2 Objectives

After going through this unit, you will be able to

- understand the meaning of decision tree,
- describe about the importance of decision tree.

3.3 Decision Tree analysis

A decision tree is a graphic display of various decision alternatives and the sequence of events as if they were branches of a tree. In decision tree analysis probabilities can be introduced into the analysis of complex decisions involving many alternatives and future conditions which are unknown but can be specified in terms of a set of discrete probabilities or a continuous

probability distribution .It is a useful tool in making decisions concerning investments, the acquisition or disposal of physical property, project management, personnel and new product strategies.

The term *decision tree* is derived from the physical appearance of the usual graphic representation of this technique. A decision tree contains not only the probabilities of outcomes, but also the conditional monetary value (or utility) values attached to those outcomes . This is the reason why decision trees are used to indicate the expected values of different actions .

Decision trees make use of standard symbols:

- Squares symbolize decision points, where the decision maker must choose among several possible actions. From these decision nodes, one branch is drawn for each of the possible actions.
- Circles represent chance events, where some state of nature is realised. These chance events are not under the decision maker's control. From these chance nodes one branch is drawn for each possible outcome.

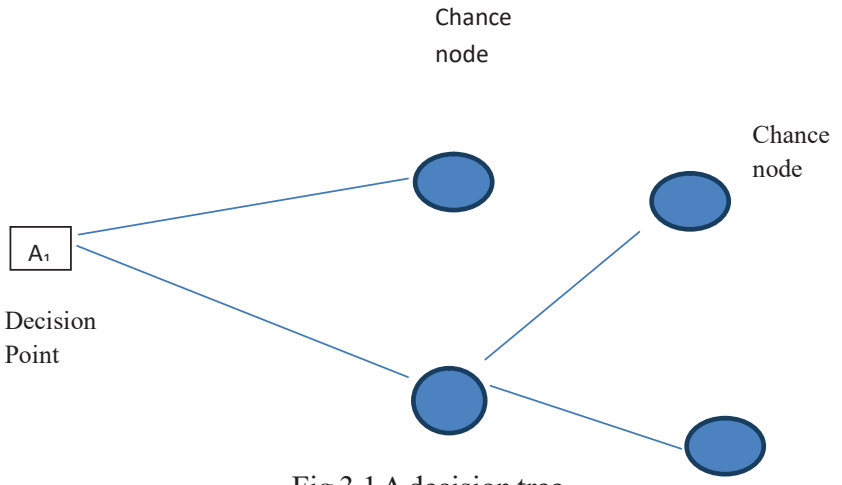


Fig 3.1 A decision tree

Check Your Progress

1. What is a decision tree?
2. State two advantages of decision tree .
3. Give two limitations of decision tree.

Various elements of the decision process like decision alternatives, states of nature, probabilities attached to the states of nature are indicated with the help of a decision tree. Branches coming out of a decision point represents the alternative courses of action. At the end of each decision branch, there is a state of nature node from which chance events arise in the form of sub branches. The respective payoffs and the probabilities associated with alternative courses and the chance events are shown alongside these branches.

Decision trees are useful for representing the interrelated, sequential and multi-dimensional aspects of a decision-making problem. By drawing a decision tree, one is in a position to visualize the entire complexity of the decision problem in all its dimensions.

Since it is impossible to evaluate an immediate decision act without first considering all future outcomes that result from this decision, one begins the analysis at the end of the tree. The last decision point is of primary importance to us. This point is analysed and decision taken which yields optimal EMV and then roll back to the last but one decision point , make the same EMV analysis for decision and roll back to the preceding decision point . The rolling back process continues till the initial decision point is reached.

3.4 Advantages and Disadvantages of Decision Trees

The following points highlights some advantages and disadvantages of Decision Trees

Advantages of the Decision Tree Approach

1. It structures the decision process and helps decision making in an orderly, systematic and sequential manner.
2. It displays the logical relationship between the parts of a complex decision and identifies the time sequence in which various actions and subsequent events would occur.
3. It communicates the decision-making process to others in an easy and clear manner, illustrating each assumption about the future.

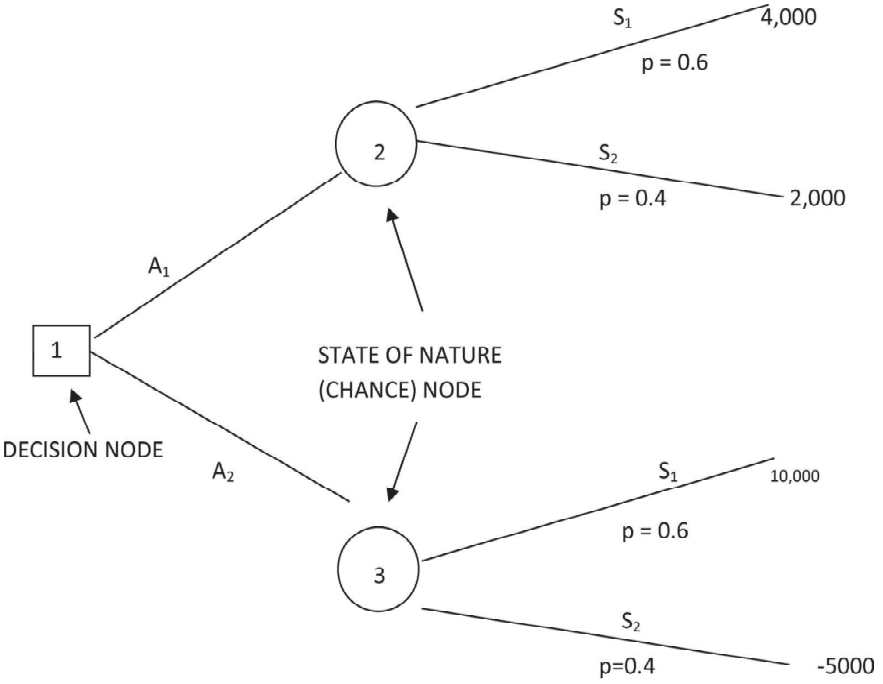
Limitations of Decision Tree Approach

1. The tree diagram become more and more complicated as the number of decision alternatives increases and more variables are introduced.

2. It analyses the problem in terms of expected values and thus A gives an average valued solution.
3. Quite often inconsistency arise in assigning probabilities for different events.

Example 1: Suppose a decision making problem is represented by the following table. -5000

States of Nature	Probability	Alternative Actions	
		A_1(produce 25 units)	A_2(produce 75 units)
S_1(High demand)	0.6	4,000	10,000
S_2(low demand)	0.4	2,000	-5000



(Fig 3.2)

The decision tree for the above problem is shown in Fig 3.2

Example 2: A trading company of Mumbai is considering expansion of its activities and planning to open a marketing office at Kolkata to boost the sales in North East. It is to be decided whether to operate from the existing office at Mumbai and cover the area by frequent travelling or else establishing the office at Kanpur. The connected pay-offs and probabilities of two alternatives are as under:

Alternatives	States of nature	Probability	Pay-off(Rs in lakhs)
A. Operate from Mumbai	(i) Increase in demand by 30%	0.60	50
	(ii) No appreciable change	0.40	5
B. Open office at Kanpur	(i) Increase in demand by 30%	0.70	40
	(ii) No appreciable change	0.30	-10

Help the company to take proper decision.

Solution: Expected pay-off for alternative A = (0.60x50)+(0.40x5)=Rs 32 lakhs

: Expected pay-off for alternative B = (0.70x40)+(0.30x (-10))=Rs 25 lakhs

The expected pay-off for alternative A being higher, it is advisable to operate from Mumbai .

As seen from the pay-offs new office at Kolkata would entail certain expenditures which may not be justified if the sales do not pick up . Hence calculated risk may be taken to operate from Mumbai only.

Example 3: A businessman has two options, either to invest in project A or B but due to the paucity of capital he is unable to undertake both of them simultaneously. Project A requires a capital of Rs 30,000and project B 50,000. Survey reports show high, medium and low demands with corresponding probabilities of 0.4, 0.4, and 0.2 respectively for project A and 0.3, 0.4 and 0.3 for project B . Net profit from investment A are Rs 75,000, Rs 55,000 and Rs 35,000 and corresponding figures for project B are likely to be Rs 100,000, Rs 80,000 and Rs 70,000 for high , medium and low demand respectively. What decision should the businessman take? Decide by constructing an appropriate decision –tree.

Solution:

Expected net profit for project A=0.4x75, 000+0.4x55, 000+0.2x35, 000-30,000

= Rs 29,000

Expected net profit for project B= 0.3x100, 000+0.4x80, 000+0.3x70, 000-50,000

= Rs 33,000

Since the expected net profit for project B is more than A, the businessman should invest in project B.

Stop to Consider

Decision theory is a systematic approach for decision making depending on the quality and quantity of information available to him or utilised by him.

There are three types of decision — making environments: certainty, uncertainty and risk.

Various elements of a decision-making process include decision alternatives, state of nature, payoff, pay-off table and opportunity loss table.

Decision tree is the graphical presentation for displaying acts and events in a decision problem in the form of a tree diagram.

3.5 Summing Up

A decision tree – A pictorial representation of a decision process, indicating alternatives and their associated probabilities; drawn in form of a tree , indicating root(the decision point) and branches (courses open , strategies, available and chances obtained with conditional payoff)

Decision Nodes represent Features/Attributes: Decision nodes represent the features/attributes based on which data is split into children nodes.

A decision tree has three key parts: a root node, leaf node and branches

3.6 Key Terms

Decision: It is a conclusion reached after due consideration.

Decision tree: A graphic display of various decision alternatives and sequence of events as if they were branches of a tree.

3.7 Model Questions

A. Multiple Choice Questions

	ColumnA		Column B
1.	Decision tree	A.	Laplace Criterion
2.	It is helpful for decision maker	B.	Is a graphic display of various decision alternatives
3.	It is based on what is known as the principle of insufficient reason.	C.	Opportunity loss table

B. Fill in the blanks

1. _____is the graphical presentation for displaying acts and events in a decision problem in the form of a tree diagram
2. _____ coming out of a decision point represents the alternative courses of action.
3. _____represent the features/attributes based on which data is split into children nodes.

C. State whether True or False

1. A decision tree is highly useful to a decision maker in multi –stage situations.
2. A decision tree has three key parts.

D. Match Column A with Column B

	Column A		Column B
1.	It is an element of decision process.	A.	The alternative courses of action.
2.	Square symbolizes	B.	Decision making under certainty
3.	Branches coming out of a decision point represents	C.	Decision points

E. Short Answer Questions

1. What are decision nodes?
2. What is a decision tree?
3. What do square represent?
4. What do circles represent?

F. Long –Answer Questions

1. Write a note on the different symbols used in decision tree.
2. What is a decision tree? Explain its advantages and limitations.
3. Give example of a decision tree.

3.8 References and Suggested Readings

Hazarika, P.L. Essential Statistics for Economics and Business Statistics.New Delhi. Akansha Publishing House, 2012

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Gupta ,S.P .Statistical Methods .New Delhi . S. Chand and Sons , 2005

Hooda , R.P . Statistics for Business and Economics. New Delhi: Macmillan India Ltd 2002

Sharma, Anand. Statistics for Management, Himalaya Publishing House, Geetanjali Press Pvt . Ltd, Nagpur

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