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OPERATIONS RESEARCH & COMPUTER IN BUSINESS

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BLOCK 1: UNIT-I

OPERATION RESEARCH

Unit Structure:

1.1 Introduction

Decision making in every sphere, it may be in business or social environment, is a very challenging task in this competitive environment. The uncertainty of the future, nature of social interaction, competition in all fields of development increases the level of difficulty of decision making in managerial level. In the changing environment of knowledge and technology in twenty first century, optimization of structured problems at operational level of organizations is very much required. The focus and attention of the organization are on broader tactical and strategic issues. The need of operation research techniques arises here to meet these problems effectively and providing best decisions to fit in this advancing global age.

Operations research as one of the quantitative aids to decision making offers the decision maker a method of evaluating every possible alternative by using various techniques to know the potential outcomes. In solving real life problem, the decision maker must proceed with both quantitative as well as qualitative approach. The evaluation of all these alternatives is very difficult in terms of time consuming and nature of complexity of information that must be processed. When there is a lack of qualitative factors decision makers usually turns to the quantitative approaches and use computers to arrive at the optimum solution of the problems including almost all the alternatives. The essence of operation research approach lies here. Operation research methods helps in decision making by studying the various methods and applying them properly in solving the problems.

Operations Research (OR) can be defined as a branch of learning that helps in making better decisions in complex systems by the application of a set of advanced analytical methods. It includes theories, some theorems and techniques of mathematics, statistics that is used for

problem solving. Applications of OR techniques spread over various fields in engineering, management and public systems.

Basically Operation Research is the field of study where we optimize the performance under some given constraints. It has its applications in business, manufacturing and in different public systems. The application of OR is extensively increased in these areas with greater accuracy. It includeslinear programming and its extensive, i.e. formulation of linear programming, solution of linear programming through graphical method, simplex method, duality of linear programming problems. It also includes the application of operation research in the field of transportation problems, assignment problems.

1.2 Objectives

- Attainment of Optimization i.e. to do the things best under the given circumstances, in each step.
- Including minimization of cost
- To improve the productivity and develop more effective approaches to the programming
- Constructive Decision making

1.3 Definition

Operation research (OR)is the branch of science that deals with the application ofsystematic, scientific advance analytical methods for analysis of complex systems and problem solving that helps in making better decisions at the managerial level of an organization.

Some of the definitions of Operation research are

"OR is a scientific method of providing executive departments with a quantitative basis for decision regarding the operations under their control." --Morse and Kimbal(1946)

"OR is a scientific method of providing executive with an analytical and objective basis for decisions." --P.M.S. Blackett(1948)

"OR is the application of scientific methods, techniques and tools to problems involving the operations of systems so as to provide these in control of the operation with optimum solutions to the problem." --Churchman, Acoff, Arnoff(1957)

"OR is a management activity pursued in two complementary ways- one half by the free and bold exercise of commonsense untrammeled by any routine, and other half by the application of a repertoire of well established precreated methods and techniques. " - -Jagajit Singh(1968)

" OR is an applied decision theory. It uses any scientific mathematical or logical means to attempt to cope with the problems that confront the executive when he tries to achieve a through going rationality in dealing with his decision problems." --Miller and Starr

" OR is a scientific method approach to problem solving for executive management"

-H.M. wagner

" OR is an aid for the executive in making his decisions by providing him with the needed quantitative information based on the scientific method of analysis." --C. Kittel

" OR is the application of scientific methods to problems arising from operations involving integrated systems of men, machines and materials. It normally utilizes the knowledge and skill of an inter –disciplinary research team to provide the managers of such systems with optimum operating solution." --Fabrycky and Torgersen

" OR is an experimental and applied science devoted to observing, understanding and predicting the behaviour of purposeful man -machine systems and OR workers are actively engaged in applying this knowledge to practical problems in business, government and society" --OR Society of America.

1.4 Origin and Development

Every development has a history. Operation research has its origin since world war II. During world war II there was a critical need of manage scarce resources. The term 'operation research' was coined as a result on military operations during the war. During war, strategic and tactical problems in various military operations were arise, that were much complicated to address, a group of specialists from various disciplines such as of Mathematics, Statistics,

probability theory, Economics, Engineering, Behavioral and physical science were formed a special unit within the armed forces to deal with such strategic and tactical problems of various military operations. During world war II, for the first time, the military management of UK used this team of scientists to address the problems related to air and land defence of their country. As resources were limited, most effective utilization of them were needed.

The team was headed by Prof. P.M.S. Blackett and was attached to the Radar Operational Research Unit. Also they were assigned to analysed the coordination of radar equipment at gun sites. The idea behind employing this group of experts from several inter-disciplinary fields of knowledge for scientific research into strategic and tactical military operations was to formulate specific proposals and plans for helping the military forces to arrive at the decisions on optimal utilization of limited resources and efforts and implement the decisions effectively. This practice gave the military forces of United Kingdon a very positive result. Following which the term Operation Research was coined.

After UK, military forces of United States of America also applied the same process. The US team of scientist became successful in various aspects through their research including new fight pattern, sea mining, and effective utilization of electronic equipment.

After the success in world war II, this group of scientists, who had been active in the military operation research groups, made efforts to apply the operation research approach to the problems of civilians, business fields, research and development etc.

Due to the economic and industrial boom by the end of the war, the industrial sector also inspired with the task of operation research and were decided to apply the operation research technique for the solutions to their complex, executive –type problems through Operation research. The main problem with the industrial sector was to gain maximum at a minimum cost and to choose the appropriate method forachieving this goal. In response to this the first method was developed by the American mathematician George b. Dantzig in 1947 known as simplex method of linear programming problem. Since then , a number of techniques were developed in this field.

Nowadays, the applications of operations research is noticed in almost all spheres of knowledge including Management, Business administration, Transportation system, city planning, hospitals, financial institution, airlines, railways etc. These sectors recognized the usefulness of the application of OR in improving the efficiency of their works. The analytic

power provided by the high -speed computers made it possible to apply many operation research techniques in decision theory.

During b1950s, there was a noticeable progress in the application of OR (operation research) for civilian activities as well as in professional development and education in OR. Many universities and colleges including schools of engineering, business management and administration, applied mathematics, Statistics, economics, computer science, library science etc. introduced OR in their curricula.

In India, existence of OR was established in 1949 when the Regional research Laboratory, Hyderabad constructed an unit of OR for planning and organizing research in their organization. Parallaly, Prof. R.S.Verma also set up an OR team at Defence Science Laboratory to solve the problems of storing, purchasing and planning. In Indian Statistical Institute, Kolkata, OR was introduced by the great statistician Prof. Mahalanobis in 1953 to solve the problems relating to national planning and survey.The Operation Research Society of Indiawas found in 1957 and the journal OPSEARCH was published from1964. India along with Japan joined in the International Federation of Operational research Societies (IFORS) in 1964whose headquarter is in London.

In 1966, PERT (Program Evaluation and Review Technique) and CPM 9critical Path method) were developed in OR technique as efficient tools for scheduling and supervising the complex and high expensive, sensitive projects. During 1960s different OR groups were formed in several organizations. New techniques were developed and some of existing techniques of OR were upgraded. Different educational and professional development programmes were introduced at all levels around the world. The American Institute of Decision sciences was formed in 1967 to promote, develop and apply quantitative approach to functional and behavioral sciences. Its journal *Decision Science*was started from1970.

With multi-disciplinary character and application in varied fields, OR has a more potential applications in the coming days. In application of Operation research, the mathematical model of the problem is formulated first which is the simple representation of the problem. Then an optimal solution for the problem is found. The contribution to the sectors of hospital management, energy conservation, environmental pollution etc. is an indication of the ability and efficiency of OR towardsimprovements of different sectors relating to the social welfare and areas of global need.

Check Your Progress

1. Operation research approach is

i) multi-disciplinary ii) scientific iii) intuitive iv) all of the above

Answer: (i)

2. A model is

i) an essence of reality ii) an approximation iii) an idealization

iv) all of the above.

Answer: (iv)

3. Give a brief description of operation research.

4. Write a note on the origin and development of operation research to reach in today's position.

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BLOCK I : UNIT-II

Phases of Operation Research (OR), Characteristics of OR model, Role of OR in managerial decision making, Use and limitations of OR

Unit Structure:

2.1 Introduction

As an interdisciplinary discipline, operation research deals with the application of scientific methods for making valid decision with proper allocation of limited resources. It provides a rational basis for decisions making in the absence of complete information. To attain this goal, operation research gives utmost importance in analysing the operations as a whole. To make a concrete and valid decision, operation research has to implement scientific methodologies in different stages of operation. This proceeds to different phases, which can be categorised into three broad phases.

To solve a particular problem, operation research provides the logical and sequential steps that the operation research personal applies them with the knowledge. For this, the phenomena of the problem have to be studied and collect all necessary information and formulate the statement of the problem. Different constraints involved, alternative procedures for the solution if exists, relations between studied variables, optimality of the solution etc. have to be studied carefully.

2.2 Objectives

- To improve the productivity through proper collection of data, selecting appropriate model and develop more effective approaches to the programming
- Constructive decision making
- To describe the applications of operation research and its limitations.

2.3 Phases of Operation research

The approach of scientific methods to build decision models in operation research have some phases, based on which, it helps in taking decisions adequately. There are three basic phases of the scientific method on which operation research is based on.. They are-

1. Judgement Phase,

2. Research Phase,

3. Action Phase.

1. Judgement Phase: This is the first phase of operation research and it includes –

- Identification of the problem.
- Selection of an appropriate objective and values of various values related to the objective..
- Selection and application of the appropriate scale of measurement i.e. deciding the measurements of effectiveness.
- Formulation of an appropriate model that describes the problem unambiguously, abstracting the essential information, so that the objective or goal of the study must be attained. Care should be taken to attain a right decision.

- 2. **Research Phase:** The largest and the most important phase. It includes
  - observations and data collection for a better understanding of the problem. The process of data collection must be strictly economical and strictly relevant to the objectives and the hypothesis. Skilled and special investigators like social investigators, demographic investigators, paramedical technicians, accountants should be employed for data collection team.
  - Formulation of the hypotheses and mathematical model. The hypotheses must be relevant to the objective of the study of the operation research. The mathematical model includes three important basic features—(i) Decision variables and parameters, (ii) Constraints or restriction and (iii) an objective function.
  - Observation and experimentation to test the hypotheses on the basis of additional data collected. In this phase, the decision variables are computed which optimize the objective function and obtained the optimal solution.
  - Analysis of the available information and verification of the hypotheses using pre-established measures of desirability.
  - Vaticinate the various probable results from the test of hypotheses and thus the validity of the model is established. For longer application of a model, it must be updated time by time by considering the past, present and future specification of the problem.
  - Generalisation of the result after validation and consideration of alternative methods if necessary. The tested results of the model are now can be implemented to work.

3. **Action Phase:**

This phase consists of making recommendations for implementing the decision arrived in phase 2 by the authority or the person appointed by the authority who can implement the decisions to work. This phase is usually executed by the operation research expert and the persons who are responsible for managing the system. This team is aware of the situation in which the problems occurred, the objectives and assumptions of the model constructed for the problem.

2.4 Characteristics of Operation Research Model

Operation Research has some specific characteristics(features). The main characteristics of operation research are—

1. Inter-disciplinary team approach: In Operation Research the optimum solution of an assignment is attained by the combined effort of a team of scientists selected from various disciplines such as Mathematics, Statistics, Economics, engineering, Physics etc. As for example, to investigate the inventory management in a factory, work of an engineer is required as functions of different items of the stores are known by the engineer as well as mathematician or statistician will also be required and so on. A mathematician or a statistician can apply their expertise in the plant after getting the physical implications of the plant from an engineer. Otherwise the solution may not be as much adequate to apply. The work become inter-disciplinary and expertise of each member of the Operation Research team is required for a better solution of the problem of study. It brings together a diverse disciplinary point of view and thus makes available different research techniques to attain the problem. Hence, operations research is characterized by combinations of disciplines on research teams and by the use of varied research methods.

2. Wholistic Approach to the system: In addressing the problems, the team of Operation Research tries to find out the best decisions for the organisation. The nature of the organisation does not matter for the Operation research team.

It tries to find out whether the behaviour of any part of a system has any effect on the behaviour of the system as a whole To run a system properly, it is very essential to interact between parts. It is not the actions of any single part of the system, that determines how well the system will perform.

For maintenance problem in a factory, Operation Research team tries to find out how it affects the production department of the organisation as a whole and if possible, its consequences on the other departments. It can also try to find out how the effect on this particular organisation in turn affects the industry as a whole and whether the effects are significant or not.

3. Imperfectness of Solutions: Operation Research approach offers the best possible solutions to the problems but cannot obtain perfect answers to our problems. It can improve the quality of the solution.

4. Scientific research Procedures or Methodological approach: Operation Research applies scientific research procedures during its execution to reach the optimum solution. It observes and defines the problem, formulates, tests the hypotheses and then analyse the results of the test obtained. The systematic and scientific way to address the problems denies any bias in the output.

5. To optimize the total output: The Operation Research procedures optimizes the total output by maximizing the profit and minimizing the cost or loss with respect to some constraints. For this, a measure of desirability is defined based on the objectives of the organization. The measure of desirability is then used to compare the alternative courses of action with respect to their outcomes.

2.5 Role of Operation Research in Managerial Decision Making:

Operation research is an approach to solve complex problems or situations through analytical procedure that helps the management in taking decision appropriately. It increases the efficiency of managerial decisions of an organization. It helps in taking decisions in the situation when the conditions are uncertain and when the differing objectives are in conflict with each other. It provides the adequate quantitative information to the manager so that a valid and appropriate decision can be taken for the organization. Operation research divides the whole work into some sub-section and executes the same through different experts in respective areas, where each sub-section is inter related. This makes better co-operation among the persons concerned and a better result becomes visible. This results in an easier and better environment to take appropriate decision at managerial level.

Some of the advantages of Operation Research approach in business and management decision making can be classified as follows:

1. Better Control: The approach of Operation Research gives stress on executing the more complex problems of an organisation or a business house and properly address the matter. This way a better control is achieved. This is possible because operations research provides a basis including standards of performance and ways to measure productivity. Reporting deviations from standards enables managers to identify problem areas and to take corrective measures.

2. Better Co-ordination: The Operation Research based planning model are like a vehicle for co-ordinating in decisions making with the limitations imposed. Operations research analysis combine together the objectives of different departments. For example, operations research coordinates the aims of the marketing department with the schedules of the production department. Thus better co-ordination among different department or sub-sections is established.

3. Better System: Operation research technique works on analysis to provide support in decision making such that whether a new site or warehouse can be established or not with all details. Also it can further developed the system to a cost effective one by finding optimal solutions to the problems arrived.

4. Better Decisions: The Operation Research provides support in making better decisions at managerial level through different procedures. Sometimes, in business houses, arises so complicated situation, that human mind can not think of assimilate all the important factors without the help of operation research. This way Operation research helps in making better decisions in a business house. The mathematical models of operations research permits

people to analyze a greater number of alternatives and constraints than would usually be possible, if they were to use only an intuitive approach. Using operations research, it is easier to analyze multiple alternatives, which results in greater confidence in the optimal choice.

5. Increased Business Productivity: Application of Operation research in business houses gives the opportunities to improve the productivity of the business through the mathematical formulas used in operations research. As these formulas offer a greater number of optimal choices of inventory mix, plant machine utilization, factory size, manpower planning, implementing new technologies etc.

Uses of Operation Research:

In recent years, the application of operation research has spread into different areas of development of a region. The different fields of application of operation research are as follows.

1. Agriculture: In agricultural field, optimum allocation of land to various crops and optimum distribution of water and other essential goods of cultivation is necessary so that the demand of food in accordance to the increased population can be meet. The increased demand of food production with limited resources is a general scenario of developing countries. So, there is a need of making proper policies to solve this problem and the need of operation research arises here.

2. Finance and Accounting:

The operation research techniques can be applied to the following areas of finance and accounting:

i)To maximize the per capita income of a country with the limited resources

ii)To determine the best replacement policies for the industries.

iii) To determine optimum profit plan for the industries.

i) In capital requirements, cash flow analysis.

ii) Credit policies

iii) Investment policies

iv) Public accounting etc.

3. Marketing:

The operation research techniques helps the Marketing manager in deciding:

i) to minimize the per unit sale price.

ii) to minimize the transportation cost for maximum profit and to achieve this in finding market for sale of product.

iii) size of inventory collection for future need.

iv) advertizing media with minimum cost, time, manpower so that profit can be maximize.

v) export- import planning

vi) best time to launch a new product.

4. Production Management:

The operation research techniques helps the production management in

i) finding the location and size of new plant, distribution centres, retail outlets.

i) finding number and size of the items to be produced.

iii) framing logistic sequence of production run by proper allocation of machines.

iv) deciding optimum product mix.

5. Personnel Management:

The areas of personal management where operation research can be applied for smooth running of a business house are as follows:

i) Manpower planning such as

- in appointing the most suitable person on minimum salary,
- in deciding the age of retirement for employees
- in determining the number of persons to be appointed on full time basis or contractual as required by the workload.

ii) Salary/ Wage management.

- In wage balancing based on skills of the employees,
- If required, in negotiation of salaries.

iii) In planning the training programmes for employees to enhance them with skills as required by the nature of the work.

iv) In designing the more effective structures of the organization.

6. General Management:

The technique of operation research can be fruitfully applied in

- Controlling quality more effectively.
- Project management, Strategic Planning.
- Decision making

7. Government

- Economic planning ,
- social welfare planning,
- planning to develop education and health sector,
- Optimum allocation of natural resources,
- energy distribution,
- proper planning in agricultural
- sector for more production,
- Industrial sector,
- planning of urbanisation,
- population control,
- Pollution control,
- Making more structured defence system etc.

2.6 Limitations of Operation Research:

Operation Research, besides its various opportunities, has some limitations also. The reason behind these limitations are mainly due to unawareness of the decision makers about their role, some institutional or organisational matters/ concepts while constructing

the model and some natural behaviour. Some of the limitations of Operation Research are as follows:

- **Higher cost:** Due to the nature of approach, application of Operations research costs high. For detailed survey, thorough analysis, professional assistance including software etc. a high amount of fees have to be paid.
- **Selection of model:** Sometimes the adopted model do not represent the realistic situations on the basis of which decisions must be taken.
- **Technology dependent:** The entire approach of operation research requires up-to-date technology which is based on computer. Huge amount of data set have to be analyzed. So any failure or missing of records affects the concerned organization.
    - **Limitations of Adopted Model:** Limitations of the model adopted also adds up to the limitation of the operation research approach (incurred to the OR), if the decision maker is not aware of it and do not take corrective measures in time.
    - **Dependence on estimates:** During the process of analysis, one has to rely on estimates in operations research. So an incorrect estimate may lead to a wrong decision
    - **Dependence on experts:** To perform the activities of operation research, a team of experts from different fields are required. A dependence on experts gives the operation research a limitation.
    - **Incalculable factors:** Some times during the process of operation research, some un measurable factors arise. In such situations effectiveness of solutions decreases.

Check Your Progress:

1. For analysing a problem, decision makers should normally study
i) its qualitative aspects ii) its quantitative aspects iii) Both (i) and (ii)
Ans: (iii)
2. Decision variables are
i) controllable ii) uncontrollable iii) parameters
Ans: (i)
3. Managerial decisions are based on
i) an evaluation of quantitative data ii) the use of qualitative factors
iii) numbers produced by formal models iv) all of the above
Ans: (iv)

4. Explain how and why operations research methods have been valuable in aiding executive decisions
5. Describe briefly the applications of operation research in managerial decision making. Give suitable examples from different areas.
6. What are the advantages and limitations of operation research? Compare.

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BLOCK I : UNIT-III
Linear Programming Problem

Unit Structure:

3.1 Introduction

Resources are limited in nature. To achieve the desired goal or objective with this limited resources, management of each organisation has to take decision on the optimal use of these resources. In this context, linear programming technique of operation research proves to be of great help in planning and decision making process at managerial level.

The linear programming technique of operation research is one of the most widely applied technique for optimal use of limited resource to reach maximum profit at minimum cost. It is a mathematical approach to allot limited resources to different heads in an optimum manner. It deals with the optimisation of a function of variables known as objective function subject to a set of constraints which are nothing but a set of linear equations and/ or inequalities. The objective function may be of maximisation of profit, production capacity of a company or minimisation of cost, etc. related to any other measure of effectiveness. The constraints are some restrictions imposed by different resources such as market demand, availability of raw material etc. The objective function and the constraints must be linear in nature.

Linear programming was first introduced by the Russian mathematician Leonid Kantorovich in the field of manufacturing schedules and by American economist Wassily Leontief in the field of economics. But it was applied at a large scale during world war II by the American mathematician George Dantzig and his associates in 1947 to solve military planning problems on a project of U.S. Air Force. Afterwards, this approach is widely applied in

business and industrial sector. In 1950, GeorgeDantzig developed 'Simplex method', the most powerful mathematical tool ,to solve the linear programming problems.

3.2. Objectives:

The main objective of linear programming is to obtain the most optimal solution to a real life problem with respect to the limited resources i.e. Obtaining optimum solutions, that is maximizing the profit and minimizing the cost,satisfying all the constraints.

3.3. Meaning of Linear Programming problem:

Linear Programming is a mathematical procedure for determination of optimum schedule of interdependent activities in view of limited resources, which areproduced in the form of a set of linear inequalities known as constraints.

In linear programming, the term "linear" indicates that the objective function, that is the function to be maximized/ minimized, and the constraints represented by a system of linear inequalities are of degree one. In brief, the word 'linear' indicates that all the relations involved in a particular problem are linear. The word "programming" indicates that the planning of activities are arranged in such a manner that optimum results with limited resources can be achieved. Shortly, the word ' programming' refers to the process of determining a particular plan of action from amongst various alternatives. A programme is optimal if it maximizes or minimizes output, profits / cost of a business house/ organization.

Various methods are provided in operation research to solve these problems. In brief we can present linear programming as a simple technique where we depict complex relationships through linear functions and then estimate the optimum values.The simplest method to apply optimization process is the linear programming. The optimization process ranges from simple problems to complex problems.

To understand what linear programming is, some prior knowledge of some terminologies related to linear programming is required. Some of the terms, used in general, in linear programming are as follows:

1. ***Decision variables:*** The unknown quantities which are expected to be determined in the form of an output of the linear programming.

The necessary requirement for the variables are

i) The variables should be inter-related in relation to consumption of resources.

ii) The relation among the variables must be linear.

2. Constraints:Constraints are the restrictions or limitations imposed by the problem characteristics i.e. on the total amount of a particular resource which is required for carrying out the activities. These constraints are written in the form of mathematical equation in a linear programming problem.

3. ***Non-negative constraints:***In Linear Programming decision variables must be positive irrespective of whether the objective function is to maximize or minimize the net present value of an activity. The non-negative constraints can take the value zero or any positive number.

4. ***Objective function:***The objective function of a linear programming is the mathematical equation of decision variables , which has to be either maximize or minimize according to the nature of the problem subject to a set of constraints. It determines the amount that would be contributed by each decision variable to the net present value of a project or an activity.

The above terms are explained with the help of an example below:

Maximize P= 3x+2y----------(1)

Subject to —

$3x+5y \geq 15$----------(2)

$2x+3y \leq 6$----------(3)

$x, y \geq 0$----------(4)

Here, x, y are the decision variables.

(1) represent the objective function , which have to maximize.

(2), (3), and (4) represent the constraints, where (4) is the non-negative constraint.

***Assumptions of Linear Programming*:**

1. Linearity: This is the basic assumption of Linear programming. The amount of each resource used and its contribution to maximizing the objective function must be proportional to the value of each decision variable. For example, if profit of making a unit of a commodity is Rs.25, then production of 30 units of that commodity will produce Rs. (25x30)= Rs. 750 as profit.

2. Additivity: According to this assumption, the total profit earned from the sale (consumed) of the products must be equal to the sum of the profits (resources used) earned separately from the products.

3. Certainty: The availability of resources and consumption per unit of products produced from these resources are known with certainty. Profit contribution of a unit of the products are also known and may be constant.

4. Continuity: Decision variables are assumed to be continuous. They can take any non-negative value and satisfy the constraint.

Check your Knowledge:

1. What is linear programming?

2. Write in short about objective function , constraints and basic assumptions of linear programming.

3.4 Uses of Linear Programming Problems:

Besides its use in decision making in business and industry, linear programming has a wide application in different areas of knowledge like in managerial economics, inter- regional trade analysis, general equilibrium analysis, welfare economics, development planning and many more. Some of the applications / uses of this technique are as follows:

1.Industrial Applications: Linear programming is widely used in business and industrial sector for production planning, transportation and routing, product mix etc.

(i) *Product mix Problems:*A company can produce different products and with limited resources it is necessary to determine the quantity of each product to be produced with the knowledge of marginal contribution of the product . Thus the objective is to maximize the total contribution subject to all constraints.

(ii) *Production scheduling Problems:*To plan and schedule the production of various items, Manufacturing companies use linear programming to achieve optimization and thus maximize their profit while minimizing the total cost of production and inventory.

(iii) *Blending Problems:*In manufacturing some products, blending (mixing) of different raw materials in varying compositions is required to achieve the desired specifications. Only linear programming can do this to produce optimum result.

(iv) *Trim– loss Problems:* In paper, sheet metal and glass manufacturing companies, standard sizes have to cut to smaller sizes as per requirement of customer with minimum waste produced. In this case linear programming is the only way.

(v) Assembly-line Balancing Problems: To assemble the different components of a product in a sequence for finalizationof the product, linear programming is applied. The objective is that the assemble time should be less than or equal to the cycle time.

Management Applications:

1. Linear programming is used by the management for selection of advertising mix from different advertising media like newspaper, magazine, radio, T.V etc. so that maximum public exposure is achieved for the products of the company/ organization subject to the total advertising budget, maximum expenditure in each media type, maximum number of insertions in each media and so on.

2. To maximize the returns or to minimize the expected risk, every company allocates an amount among different investment alternatives like saving certificates, bonds, mutual funds, real estate etc. and all these can be reached with the help of linear programming technique.

3. To maximize the profit margin from each investment in plant facilities, machinery, inventory etc., different planning, of profit are made at managerial level and application of linear programming arises here.

4. *Transportation Problem:* Linear programming technique is used to minimize the transportation cost of a company / business house to maximize the profit.

5. *Assignment problem:* In allocating facilities to each one, machine or individual, engaged in the production process linear programming is used so that total time to complete the assigned job is minimized.

Miscellaneous Applications

1. Linear programming is also used in Airlines for scheduling flights, both scheduling aircraft and scheduling staff. That is scheduling the various routes of the flights and to schedule crews to the flights. In addition, linear programming is also used for ticket pricing for various types of seats and levels of services to be provided, and to determine timing at which ticket prices will change.

2. Linear programming can also be applied in Delivery services so that shipment time/ cost can be minimized.

3. To optimize the process of ordering the products from manufacturers and scheduling the deliveries from their stores, Retailers use linear programming for smoothing running of their work.

4. Financial institutions also use linear programming to determine how to mix the financial products they offer, or in scheduling the payments transferring funds between institutions.

5. In Health sector linear programming can be used to ensure the proper supplies are available when needed. Linear programming can also be used to organize and coordinate life saving health care procedures.

6. In different areas say in the field of advertisement, Insurance etc. the method of linear programming can be applied for maximum profit at minimum cost.

7. Linear programming is extensively used in farm economics where optimum allocation of crop production is required. Linear programming is used here for specification of efficient production pattern by taking regional land resources and national demand as constraint.

8. Military use of linear programming is to select an air weapon system against enemy to keep them pinned down and at the same time keeping the use of aviation gasoline at a minimum level etc.

3.5 Limitations of Linear Programming:

In spite of increasing use of linear programming methods in different fields, it has some limitations. The limitations are as follows:

1. The linearity assumption of linear programming states that the variables are linearly related, but in real life situation, such type of linearity is always not visible, whether in the objective function or in the constraints. The non-linear relations among the variables are also exists. This becomes a great limitation for linear programming procedure.

2. The linear programming method allows fractional values for the variables but in the production procedure it may not be applicable everywhere. For example, requirement of men power, number of machines cannot be in fraction. For optimality, such types of problems can be solved by integer programming.

3. The certainty assumption of linear programming is not always true. In real life situation, the values of the constraint are not always known or they are not always constant.

4. Single objective function is another important limitation of linear programming; because, conflicting and multi-objective function may exist in real life situations.

5. After construction of a specific objective function, determination of various technological, financial and other constraints is not so easy.

6. The basic assumption of linear programming states that the constraints and the inputs remains constant during the complete period of production. But in reality, they may change depending on the changes occurring in internal and external conditions.

6. Linear programme assumes that all inputs are quantifiable, which is not always true. Some costs and benefits can not be quantifiable like goodwill.

7. The linear technique do not consider the effect of changes in cost due to quantity discounts, reduction in manufacturing time / cost due to some external cause like temporary increase in prices/ costs etc.

8. Perfect competition in product and factor markets is another assumption of linear programming technique whereas such type of perfect competition is not exists in reality.

7. Constant returns is another basic assumption of linear programming technique, but in reality, there are either diminishing or increasing returns which a company/ business house experiences in production.

8. The linear programming technique is highly mathematical and complicated in nature. Proper mathematical base is required to solve such types of problems.

3.6 General Mathematical Model Of Linear Programming Problem:

The general linear programming problem with n decision variables and m constraints can be stated as follows:

Determine the values of the variables $x_1, x_2, \ldots, x_n$ such that

Optimize (maximize or minimize) $Z = c_1x_1 + c_2x_2 + \ldots + c_n x_n$

Subject to the constraints,

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n} x_n (\leq, =, \geq) b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n} x_n (\leq, =, \geq) b_2$

....

$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}\, x_n\ (\leq, =, \geq)\ b_m$

and $x_1, x_2, \ldots, x_n \geq 0$

In short, the above formulation can be expressed as follows

Optimize (maximize or minimize) $Z = \sum_{j=1}^{n} c_j x_j$ (objective function)

Subject to the constraints,

$$\sum_{j=1}^{n} a_{ij} x_j\ (\leq, =, \geq) b_i\ , i = 1,2, \ldots\ldots, m \qquad \text{(constraints)}$$

$x_j \geq 0$; j = 1, 2,.... , n (non-negativity conditions)

Here the c_j's are the coefficients representing the per unit contribution of decision variable x_j, to the value of objective function. The a_{ij}'s(positive, negative or zero) are called the technological coefficients or input – output coefficients. These represent the amount of resource, say i consumed per unit of variable (activity) x_j. The $b_i \geq 0$, for all i, represents the total availability of the i^{th} resource.

Check your progress:

1. Write briefly on the applications of linear programming.
2. What are the limitations of linear programming technique?

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BLOCK I : UNIT-IV

Formulation of Linear programming Problems, Solutions of Linear Programming, Solutions of Linear Programming By Graphic Method and Simplex Method, Artificial Variable Technique

4.1 Introduction

Linear programming problem is an optimization technique, in which the optimum solution of the given problem is obtained by solving the linear equations (constraints). To apply this technique, the real life problems have to convert to a mathematical form, otherwise the linear programming technique can not be applied. Conversion of a real – world problem to a mathematical one is consist of construction of an objective function and a set of linear inequalities known as constraints by studying the problem thoroughly.

After formulation of the problem linear programming is applied for optimum solution. There are two main methods of solving these problems in linear programming technique such as graphical method and Simplex method. If there are two decision variables then both graphical and simplex method can be used.The graphical method do not contain any complex mathematical work. For more than two decision variables there is simplex method in linear programming technique for solution.

4.2Objectives

The objectives of this chapter are

1. To convert the real-life problem into amathematical form consists of some equations that can be solved using proper methods.

2. After formulation of the problem, the objective function is either maximized or minimized, i.e. optimized, using Graphical method, Simplex method and Artificial variable technique and reached at optimal solution considering all constraints.

4.3 Formulation of Linear programming Problems:

The main steps of formulatinga problem in mathematical form, so that linear programming technique can be used for optimum solution, are as follows:

1. ***Identify the problem data:***To solve a problem, problem data should be identified so that actual values for the decision variables can be provided. To provide correct values for the decision variables, each information related to the problem under study should be examine. These quantities constitute the problem data.
2. ***Identify the decision variables*:** Identify the decision variables of the problem If there are several decision alternatives then the decision variables have to choose according to the most appropriate decision to be taken that will optimize the objective function. After choosing the decision variables, the symbolic notation for these decision variable have to be decided along with the unit of measurement.
3. ***Formulation of the Objective function:*** Determine the objective function as required. Determine whether the objective function to be minimized or maximized and accordingly convert it into a linear mathematical expression in terms of decision variables multiplied by their profit or cost contributions.
4. ***Formulation of the constraints***: Construct the set of linear inequalities known as constraints after thorough study of the problem. In constructing these inequalities the limitation of the resources should be studied well. These constraints are the conditions that the decision variables must satisfy to constitute an acceptable (Feasible) solution. The source of these constraints are the physical limitations, logical restrictions on individual variables, management imposed restrictions etc.

Example: A company has two plants and each produces and supplies two products say A and B. Each of the plants can work up to 16 hours a day. Plant I takes three hours to prepare and pack 1000 gallons of product A and one hour to prepare and pack a quintal of product B. It takes two hours to prepare and pack 1000 gallons of product A and one and half hours (1.5 hours) to prepare and pack a quintal of product B. In plant I, Rs, 15000 is required to prepare and pack 1000 gallons of A and Rs 28,000 to prepare and pack a quintal of B. Again in plant II, Rs, 18,000 is required to prepare and pack 1000 gallons of A and Rs 26,000 to prepare and pack a quintal of B. The company wants to produce at least 10 thousand gallons of A and 8 quintals of B at daily basis.

Formulate this as a linear programming model to find out how the company should manage its production so that the required amounts of the two products be obtained at minimum cost.

Solution:

Let us first summarized the given information in the tabular form as follows

Constraints/ Resources	Product		Availability
	A	B	
Preparation time(in hours)	Plant I:3hrs/ 1,000gallons	Plant I: 1hr/ quintal	16
	Plant II:2hrs/ 1,000gallons	Plant II: 1.5hrs/quintal	16
Cost of Production (Rs.)	Plant I:15,000/ 1,000gallons	Plant I: 28,000/ quintal	
	Plant II:18,000/ 1,000gallons	Plant II: 26,000/quintal	
Minimum daily production	10,000 gallons	8 quintals	

Decision Variables:

Let,

x_1= Quantity of the product A(in '000 gallons) to be produced in Plant I

x_2= Quantity of the product A(in '000 gallons) to be produced in Plant II

x_3= Quantity of the product B(in quintals) to be produced in Plant I

x_4= Quantity of the product B(in quintals) to be produced in Plant II

The objective function

Minimize (total cost) $Z= 15{,}000x_1 + 18{,}000\ x_2 + 28{,}000\ x_3 + 26{,}000\ x_4$

Subject to the constraints

(Time constraints)

$$3x_1 + x_2 \leq 16$$

$$2x_3 + 1.5\ x_4 \leq 16$$

(Daily Production constraints)

$$x_1 + x_2 \geq 10$$

$$x_3 + x_4 \geq 8$$

and $x_1, x_2, x_3, x4 \geq 0$

Example: An electronic company produces three types of parts for automatic washing machines. It purchases casting of the parts from a local foundry and then finishes the part on drilling, shaping and polishing machines.

The selling prices of parts A, B and C respectively are Rs 8, Rs. 10 and Rs. 14.All parts made can be sold. Casting for parts A,B and C respectively cost Rs. 5, Rs. 6 and Rs. 10.

The shop possesses only one of each type of machine. Costs per hour to run each of the three machines are Rs. 20 for drilling, Rs. 30 for shaping and Rs. 30 for polishing. The capacities (parts per hour) for each on each machine are shown in the following table:

Machine	Capacity per hour		
	Part A	Part B	Part C
Drilling	25	40	25
Shaping	25	20	20
Polishing	40	30	40

How many parts of each type should be produced by the company per hour in order to maximize profit for an hour's run. Formulate this as a linear programming model such that total profit of the company is maximized.

Solution: The decision variables

x_1= Number of parts of type A to be produced by the company per hour.

x_2= Number of parts of type B to be produced by the company per hour.

x_3= Number of parts of type C to be produced by the company per hour.

The profit will be,

Selling price – cost of casting -- cost of drilling – cost of shaping – cost of polishing

Since, 25 type A parts per hour can be run on the drilling machine at a cost of Rs. 20, then Rs. (20/25) = Re. 0.80 is the drilling cost of per type A part. Similarly we can find out the cost of shaping and polishing of per type A and so on.

Therefore, Profit per type A part = $8 - 5 - \frac{20}{25} - \frac{30}{25} - \frac{30}{40} = 0.25$

Profit per type B part = $10 - 6 - \frac{20}{40} - \frac{30}{20} - \frac{30}{30} = 1$

Profit per type C part = $14 - 10 - \frac{20}{25} - \frac{30}{20} - \frac{30}{40} = 0.95$

Again, on the drilling machine, one type A part consume (1/25) th of the available hour, a type B part consumes (1/40)th and a type C part consumes (1/25)th of an hour. Thus the drilling machine constraint is

$$\frac{x_1}{25} + \frac{x_2}{40} + \frac{x_3}{25} \leq 1$$

Similarly other constraints can be obtained.

Therefore, the Linear programming model of the given problem is

Maximize $Z = 0.25x_1 + 1.00\ x_2 + 0.95\ x_3$

Subject to the constraints

1. Drilling machine constraint

$$\frac{x_1}{25} + \frac{x_2}{40} + \frac{x_3}{25} \leq 1$$

2. Shaping machine constraint

$$\frac{x_1}{25} + \frac{x_2}{20} + \frac{x_3}{20} \leq 1$$

3. Polishing machine constraint

$$\frac{x_1}{40} + \frac{x_2}{30} + \frac{x_3}{40} \leq 1$$

And $x_1 + x_2 + x_3 \geq 0$

Example:A pharmaceutical company produces two pharmaceutical products A and B. Production of both products requires the same process, I and II. The production of B results also in a by-product C at no extra cost. The product can be sold at a profit of Rs. 3 per unit

and B at a profit of Rs. 8 per unit. Some of this by-product can be sold at a unit profit of Rs. 2, the remainder has to be destroyed and the destruction cost is Re 1 per unit .Forcast shows that only up to 5 units of C can be sold. The company gets 3 unit of C for each unit of B produced. The manufacturing times are 3 hours per unit for A on process I and II respectively and 4 hours and 5 hours per unit for B on process I and II respectively. Because the product C results from producing B, no time is used in producing C. The available times are 18 and 21 hours of process I and II, respectively. Formulate this as a linear programming model to determine the quantity of A and B which should be produced, keeping C in mind to make the highest total profit to the company.

Solution:

Let us first summarized the given information in the following table

Constraints/ Resources	Time required by(in hours)			Availability
	A	B	C	
Process I	3	4	-	18hrs
Process II	3	5	-	21 hrs
By-product ratio from B	-	1	3	5 units(maximum units that can be sold
Profit per unit (Rs.)	3	8	2	

The decision variables:

Let, x_1, x_2 = Unit of product A and B to be produced, respectively.

x_3, x_4 =Unit product of c to be produced and destroyed respectively.

Thus the Linear programming model (LP model) is

Maximize $Z = 3x_1 + 8x_2 + 2x_3 - x_4$

Subject to the constraints

1. Manufacturing constraint for the product A and B

$$3x_1 + 4x_2 \leq 18$$

$3\ x_1 + 5x_2 \leq 21$

2. Manufacturing constraint for by-product C

$x_3 \leq 5$

$-3\ x_2 + x_3 + x_4 = 0$

And $x_1, x_2, x_3, x_4 \geq 0$

Check your knowledge:

1. Mathematical model of linear programming problem is important because

a) It helps in converting the verbal description and numerical data into mathematical expression.

b) Decision makers prefer to work with formal models

c) It captures the relevant relationship among decision factors.

d) It enables the use of algebraic technique.

2. A constraint in an LP model restricts

a) Value of objective function. b) Value of a decision variable

c) Use of the available resource. d) all of the above.

3. Which of the following is not a characteristic of LP model

a)Alternative courses of action b) An objective function of maximization type.

c) Limited amount of resources. D) non-negativity condition on the value of decision variables.

Check your knowledge:

1.

A tape recorder company manufactures models A,b and C which have profit contributions per unit of Rs. 15, Rs. 40 and Rs. 60, respectively. The weekly minimum production requirements are 25 units for model A, 130 units for model b and 55 units for model C. Each type of recorder requires a certain amount of time for the manufacturing of component parts, for assembling and for packing. Specifically, a dozen units of model A require for manufacturing 3 hours for assembling and 1 hour for packaging. The corresponding figures for a dozen units of model B are2.5, 4 and 2 and a dozen units of model C are6, 9 and4. During the forthcoming week, the company has available 130 hours of manufacturing, 170 hours of assembling and 52hours of packaging time. Formulate this problem as an LP model such that profit of the company is maximized.

2.

A manufacturing company is engaged in producing three types of products A, B and C. The production department produces, each day, components sufficient to make 50 units of A, 25 units of B and 30 units of C. The management is confronted with the problem of optimizing the daily production of products in assembly department where only 100 man hours are available daily to assemble the products. The following additional information is available.

Type of Product	Profit contribution per unit ofProduction (in Rs.)	Assembly time per product (in hrs)
A	12	0.8
B	20	1.7
C	45	2.5

The company has a daily commitment for 20 units of product A and a total of 15 units of products B and C. Formulate this problem as an LP model such that the totalprofit of the company is maximized.

4.4 Solution of LPP(Linear Programming Problem) by Graphical Method:

Some important definitions:

Solutions: Theset of values of decision variables x_j (j = 1,2,...,n) which satisfy the constraints of an linear programming problem (LPP) is said to constitute solution to that LPP.

Feasible solution: The set of values of decision variables x_j (j = 1,2,...,n) which satisfy all the constraints and non-negativity conditions of an LPP simultaneously is known as the feasible solution of that LP P.

Infeasible solution:The set of values of decision variables x_j (j = 1,2,...,n) which do not satisfy all the constraints and non-negativity conditions of an LPP simultaneously is known as the infeasible solution of that LP P.

Basic solution: For a set of m simultaneous equations in n variables (n > m), a solution obtained by setting (n – m) variables equal to zero and solving for remaining m equations in m variables is known as a basic solution.

The (n – m) variables whose values did not appear in this solution are called non-basic variables and the remaining m variables are called basic variables.

Basic feasiblesolution: A feasible solution to an LPP which is also the basic solution is known as the basic feasible solution. That is, all basic variables assume non-negative values. Basic feasible solution is of two types:

a) Degenerate: A basic feasible solution having the value zero of at least one basic variable is known as degenerate solution.

b) Non-degenerate: A basic feasible solution is known as a non-degenerate if values of all m basic variables are non-zero and positive.

***Optimumbasic feasiblesolution*:**A basic feasible solution which optimizes (maximizes or minimizes) the objective function of the given LPP is called an optimum basic feasible solution.

Unbounded solution:A solution which can increase or decrease the value of objective function of the LPP indefinitely is known as unbounded solution.

4.5 The Graphical Method of Solution

After formulation of the mathematical model, the next step is to solve the problem for optimal result. A linear programming problem with only two decision variables can be solved by the graphical or geometrical method. Though in reality such small problems (i.e. with two variables) are rarely exists , the graphical method provides a pictorial representation of the solution process

and helps in having the knowledge of basic concepts used in solving large LPP. The method consists of the following steps:

Step 1: Formulate the mathematical model of the given problem.

Step 2: For non-negativity constraints $x1 \geq 0$ and $x2 \geq 0$, draw the x1 and x2 axes. This imply that the variables can lie only on the first quadrant, which eliminates the possibility of number of infeasible alternatives that lie in other quadrants.

Step 3: Draw lines for each of the constraint in the x1-x2 plan. The constraints, whether equations or inequalities, are plotted as equation.

Step 4: Identify the feasible region or the solution space that satisfies all the constraints simultaneously. For $\geq$ type constraint, the area on or above the constraint line i.e. away from the origin and for $\leq$ type constraint, the area on or below the constraint line i.e. towards the origin will be considered. The area common to all the constraints is called feasible region and is shown shaded. Any point on or within the shaded region represents a feasible solution to the given problem. Feasible region is also called *convex polygon*.

Step 5: In this step, any one of the two approaches can be used – either iso-profit (cost) line approach or extreme point enumeration approach.

To use iso-profit (cost) function line approach, plot the objective function by assuming $Z = 0$. This line passes through the origin. As the value of Z is increased from zero, the line starts moving to the right, parallel to itself. Draw lines parallel to this line till the line is farthest way from the origin(for maximization Problem). For a minimization problem, the line will be nearest to the origin. The point of the feasible region through which this line passes will be nearest to the origin. The point of the feasible region through which this line passes will be the optimal point. If this line coincide with one of the edges of the feasible region, then every point on that edge will give the same maximum/ minimum value of the objective function and will be the optimal point.

Alternatively, in extreme point enumeration approach, co-ordinates of each extreme point (or corner point or vertex) of the feasible region is to be found. Then find the value of the objective function at each extreme point. The point at which objective function is maximum/ minimum is the optimal point and its co-ordinates give the optimal solution.

Example: Solve the LPP

Maximize $Z = 2x1 + x2$

Subject to the constraints

$$x_1 + 2x_2 \leq 10$$

$$x_1 + x_2 \leq 6$$

$$x_1 - x_2 \leq 2$$

$$x_1 - 2x_2 \leq 1$$

and $$x_1, x_2 \geq 0$$

Solution:

Consider all this inequalities (constraints) as the linear equations, plot them in the x_1x_2 plane. Use the inequality condition of each Constraint to mark the feasible region as Shown In the fig.1. The feasible region is shown by the shaded area. It is to be noted that the area below the lines $x_1 - x_2 = 2$ and $x_1 - 2x_2 = 1$ is not considered for the negative values of x2, otherwise non-negativity restriction is not satisfied i.e. $x2 \geq 0$.

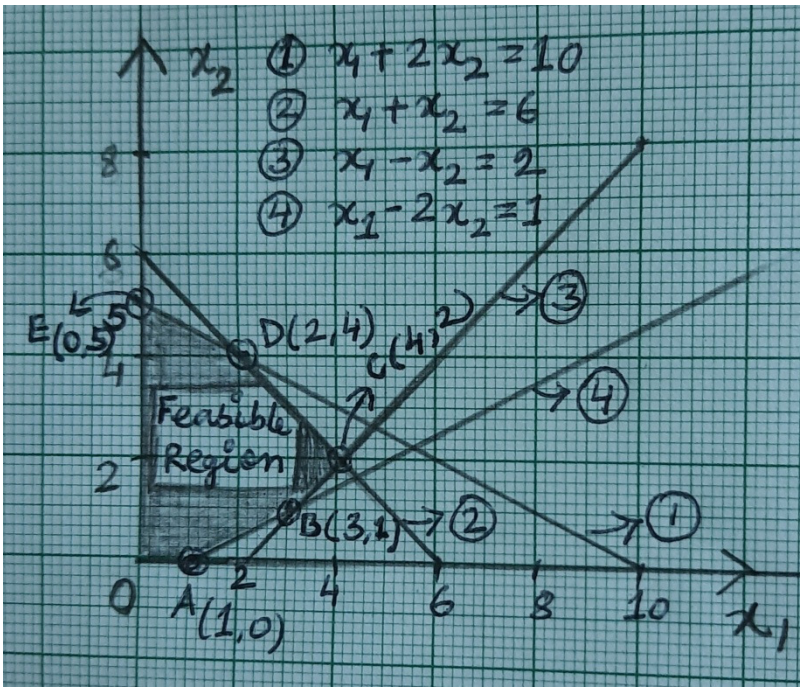

Fig. 1

The co-ordinates of the extreme points of the feasible region are,

O (0,0), A(1,0), B(3,1), C (4,2), D (2,4) and E (0,5).

Thus, the value of objective function at each of these extreme points is as shown in the table below:

Extreme Point	Co-ordinates (x_1, x_2)	Value of objective function $Z=2x_1 + x_2$
O	(0,0)	2(0) + 1(0) = 0
A	(1,0)	2(1) + 1(0) = 2
B	(3,1)	2(3) + 1(1) = 7
C	(4,2)	2(4) + 1(2) = 10
D	(2, 4)	2(2) + 1(4) = 8
E	(0,5)	2(0) + 1(5) = 5

As maximum value of the objective function Z is obtained at the point (4,2), therefore the optimal solution for the given LPP is $x_1 = 4$, $x_2 = 2$ and the optimal value is Z = 10.

Example:

Solve the following LPP by graphical method

Minimize $z = 3x_1 + 2x_2$

Subject to the constraints

$$5x_1 + x_2 \geq 10$$

$$x_1 + x_2 \geq 6$$

$$x_1 + 4x_2 \geq 12$$

and $x_1, x_2 \geq 0$

Solution:

Consider all this inequalities (constraints) as the linear equations, plot them in the x_1x_2 plane. Use the inequality condition of each Constraint to mark the feasible region as Shown in the fig.2. The feasible region is shown by the shaded area.

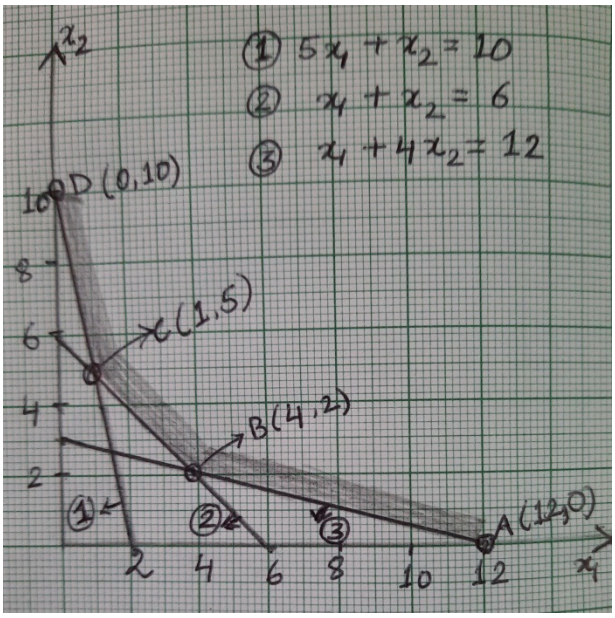

Fig. 2

The co-ordinates of the extreme points of the feasible region (bounded from below) are,

A = (12,0), B = (4,2), C = (1,5), and D = (0,10). The value of objective function at each of these extreme points is as follows:

Extreme Point	Co-ordinates (x_1, x_2)	Value of objective function $Z=3x_1 + 2x_2$
A	(12,0)	3(12) + 2(0) = 36
B	(4,2)	3(4) + 2(2) = 16
C	(1,5)	3(1) + 2(5) = 13
D	(0, 10)	3(0) + 2(10) = 20

The minimum value of the objective function Z= 13 is obtained at the extreme point (1,5), therefore the optimal solution for the given LPP is $x_1 = 1$, $x_2 = 5$ and the optimal value is Z = 13.

Example:

A firm plans to purchase at least 200 quintals of scrap containing high quality metal X and low quality metal Y. It decides that the scrap to be purchased must contain at least 100 quintals of X metal and not more than 35 quintals of Y –metal. The firm can purchase the scrap from two suppliers (A and B) in unlimited quantities. The percentage of X and Y metals in terms of weight in the scrap supplied by A and B is given below:

Metals	Supplier A	Supplier B
X	25%	75%
Y	10%	20%

The price of A's scrap is Rs. 200 per quintal and that of B is Rs. 400 per quintal. The firm wants to determine the quantities that it should buy from the two suppliers so that total cost is minimized.

Solution:

Let, x_1= Quantity of scrap (in quintals) purchased from supplier A .

x_2= Quantity of scrap (in quintals) purchased from supplier B .

The LP model is:

Minimize $Z = 200x_1 + 400x_2$

Subject to the constraints

1. Maximum purchase constraint

$$x_1 + x_2 \geq 200$$

2. Scrap containing X and y metal constraints

$$\frac{x_1}{4} + \frac{3x_2}{4} \geq 100 \; or \; x_1 + 3x_2 \geq 400$$

$$\frac{x_1}{10} + \frac{x_2}{5} \leq 35 \; or \; x_1 + 2x_2 \leq 350$$

and $x_1, x_2 \geq 0$

The constraints are plotted on the graph paper as shown in the fig. 3. The feasible region is shown by the shaded area and is bounded by the extreme points A = (100,100), B= (250, 50) and C= (50,150).

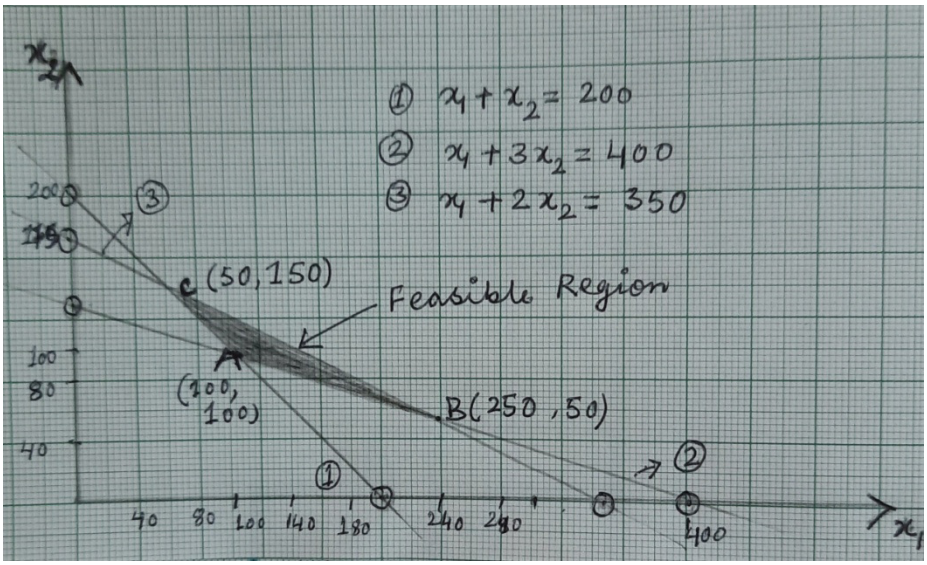

Fig. 3

The value of the objective function at each of the extreme points are as given in the table below:

Extreme Point	Co-ordinates (x_1, x_2)	Value of objective function $Z=200x_1 + 400x_2$
A	(100,100)	200(100) + 400(100) = 60,000
B	(250,50)	200(250) + 400(50) = 70,000
C	(50,150)	200(50) + 400(150) = 70,000

As Z attains its minimum value at A(100,100), the solution of the given problem is x_1=100, x_2 =100 and the minimum value of Z is 60,000.

Example:

Solve the following LPP by using graphical method

Maximize $Z = 2x_1 + 3x_2$

Subject to the constraints

$x_1 + x_2 \leq 30$

$x_2 \geq 3$

$0 \leq x_2 \leq 12$

$0 \leq x_1 \leq 20$

$x_1 - x_2 \geq 0$

and $x_1, x_2 \geq 0$

Solution:

Consider all this inequalities (constraints) as the linear equations, plot them in the x_1x_2 plane. Use the inequality condition of each Constraint to mark the feasible region as shown in the fig.4. The feasible region is shown by the shaded area.

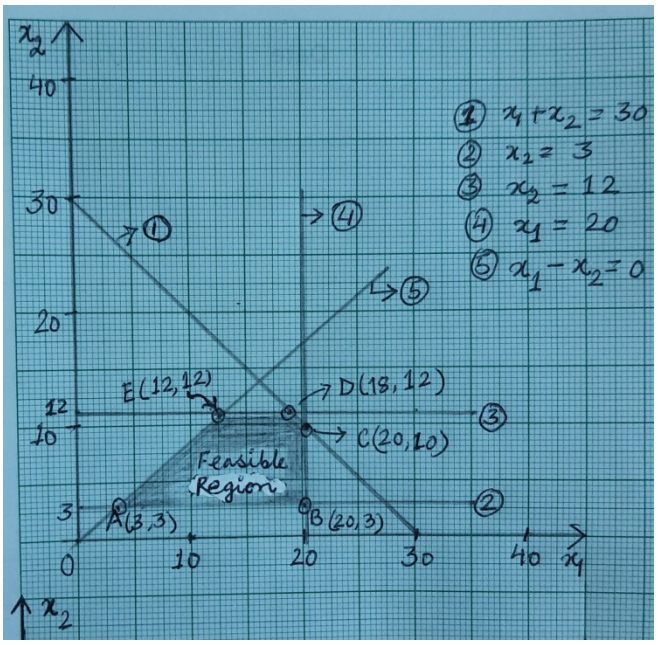

Fig. 4

The co-ordinates of the extreme points of the feasible region are,

A = (3,3), B = (20,3), C = (20,10), D = (18,12) and E = (12,12). The value of objective function at each of these extreme points is as follows:

Extreme Point	Co-ordinates (x_1, x_2)	Value of objective function $Z=2x_1 + 3x_2$
A	(3,3)	2(3) + 3(3) = 15
B	(20,3)	2(20) + 3(3) = 49
C	(20,10)	2(20) + 3(10) = 70
D	(18, 12)	2(18) + 3(12) = 72
E	(12,12)	2(12)+ 3(12) = 60

The maximum value of the objective function Z= 72 is obtained at the extreme point E= (12,12), therefore the optimal solution for the given LPP is $x_1 = 12$, $x_2 = 12$ and the optimal value is Z = 72.

1.6 Graphical Solution to Special cases in LPP:

Unbounded Solution:

In LPP, if values of one or more decision variable and the value of the objective function are permitted to increase infinitely without violating the feasibility condition, then the solution is called an unbounded solution. Some mistake in mathematical model formulation is the cause of such type of LPP.

Example: Use the graphical method to solve the following LPP

Maximize Z = 3x1 + 4x2

Subject to the constraints

$x_1 - x_2 = -1$

$-x1 + x2 \leq 0$

And x1, x2, ≥ 0

Solution:

Plot each constraint on the graph paper by treating the inequalities as equality. The feasible region is shown by the shaded region in fig 5. From the graph it is clear that, both the variablesx1 and x2 can be made arbitrarily large and the value of objective function z is also increased. Thus the problem has an unbounded solution.

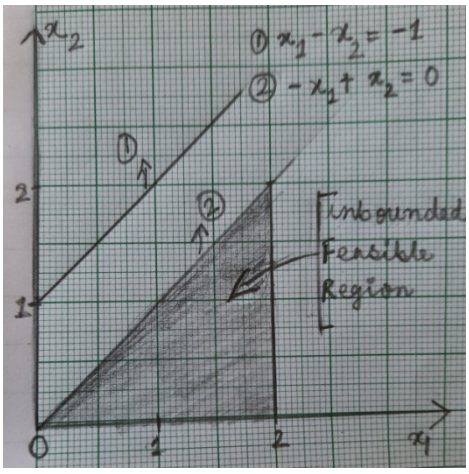

Fig. 5

LPP with Infeasible Solution :

Due to wrong model formulation, sometime, no value of the variables satisfy all the constraint simultaneously. The condition of infeasibility arises in this situation.

Example:

Maximize $Z = 6x_1 - 4x2$

Subject to the constraints

$2x_1 + 4x_2 \leq 4$

$4x_1 + 8x_2 \geq 16$

And $x_1, x_2 \geq 0$

Solution:

The constraints are plotted on the graph paper by treating the inequalities as equality as shown in fig. 6. It is clear from the graph that there is no unique feasible region to get unique set of values of variables x_1 and x_2 that satisfies all the constraints. Thus, there is no feasible solution to such type of LPP because of conflicting constraints.

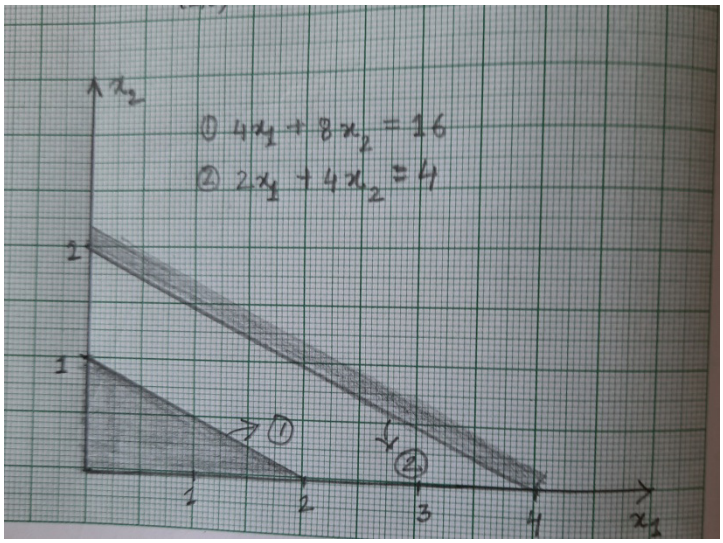

Fig. 6

Check your Progress:

Concept based questions:

1. The graphical method of LPP uses—

a) Objective function equation b) Linear equations

c) Constraint equations d) All of the above

2. A feasible solution to an LPP

a) Must satisfy all constraints simultaneously b) must be a corner point of the feasible region

c)Must optimize the value of the objective function.

3. An iso-profit line represents

a) An infinite number of solutions all of which yield the same profit.

b) An infinite number of solutions all of which yield the same cost.

c) An infinite number of optimal solutions. d) A boundary of the feasible region.

4. If an iso-profit line yielding the optimal solution coincides with a constraint line, then

a) The solution is unbounded b) The solution is infeasible

c) The constraint which coincides is redundant. D) None of the above

5. If two constraints do not intersect in the positive quadrant of the graph, then

a) The problem is infeasible b) The solution is unbounded

c) One of the constraint is redundant d) None of the above.

Answer: 1. (d), 2. (a), 3. (a), 4.(d), 5. (a)

Self Practice Problem:

Solve the following LPP graphically and state what your solution indicates:

1. Min $Z = 4x_1 - 2x_2$

Subject to $x_1 + 2x_2 \leq 14$

$3x_1 + 2x_2 \geq 36$

$2x_1 + x_2 \leq 24$

And $x_1 , x_2 \geq 0$

[Ans 1: $x_1 = 8$, $x_2 = 6$ and Min $Z = 20$]

2. . Min $Z = 20x_1 + 10x_2$

Subject to $x_1 + 2x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \leq 60$

And $x_1 , x_2 \geq 0$

[Ans 2: $x_1 = 6$, $x_2 = 12$ and Min $Z = 240$]

3. Max. $Z = 40x_1 + 100x_2$

Subject to $12x_1 + 6x_2 \leq 3{,}000$

$4x_1 + 10x_2 \leq 2{,}000$

$2x_1 + 3x_2 \leq 900$

And $x_1 , x_2 \geq 0$

[Ans 3: Multiple optimal solution with a profit value Rs. 20,000]

4. Max. $Z = 5x_1 + 4x_2$

Subject to $x_1 - 2x_2 \leq 1$

$x_1 + 2x_2 \geq 3$

And $x_1 , x_2 \geq 0$

[Ans 4: Unbounded solution]

4. A company machines and drills two castings X and Y. The time required to machine and drill one casting including machine set up time is as follows:

Casting	Machine Hours	Drilling Hours
X	4	2
Y	2	5

There are two lathes and three drilling machines. The working week is of 40 hours; there is no loss time and overtime. Variables costs for both castings are Rs. 120 per unit while total fixed costs amount to Rs. 1,000 per week. The selling price of casting X is Rs. 300 per unit and that of Y is Rs. 360 per unit. There are no limitations on the number of X and Y casting that can be sold. The company wishes to maximize3e its profit. You are required to (i) formulate a linear programming model for the problem and 9ii) solve the problem graphically. [Ans: $x_1 = 10$, $x_2 = 20$ and Max $Z = 5{,}600$]

5. A company manufactures two products A and B. Product A yields a contribution of Rs. 30 per unit and product B of Rs. 40 per unit towards fixed costs. It is estimated that sales of product A for the coming months will not exceed 20 units. Sales of product B have not been estimated but the company does have a contract to supply at least 10 units to a regular customer.

Machine hours available for the coming month are 100 and products A and B take 4 hours and 6 hours of labour respectively. Materials available are restricted to 40 units of two products while each uses one unit material per unit. The company wishes to maximize contribution. Using the graphic method, find the optimum product mix.

6. The ABCcompany has been a producer of picture tubes for television sets and certain printed circuits for radios. The company has just expanded into full scale production and marketing of AM-FM radios. It has built a new plant that can operate for 48 hours per week. Production of an AM radio in the new plant will require 2 hours and production of an AM- FM radio will require 3 hours. Each AM radio will contribute Rs. 40 to profits while an AM-FM radio will contribute Rs. 80 to profits. The marketing department, after extensive research, has determined that a maximum of 15 Am radios and 10 AM-Fm radios can be sold each week. Formulate this problem as an LP model to determine the optimal production mix of AM-FM radios that will maximize profits. [Ans: x1 = 9, x2 = 10 and Max Z = Rs. 1,160]

4.7 Simplex Method to Solve An LPP:

Linear programming problems with more than two variables are not solvable by the graphical method. Analytic solution is also not possible as tools of analysis are not suitable to handle the inequalities of the linear programming problems. For optimal solutions of such problems a method known as simplex method was introduced and developed by G. Dantzig in 1947.

For LP problems with several variables, the feasible region may not be able to graph, but the optimal solution will still lie at an extreme point of the many sided, multi dimensional figure (n-dimensional polyhedron) that represents the area of feasible solutions. The simplex method provides a systematic method for examining the extreme points (vertexes) until an optimal solution is reached. It repeats the same set of steps, i.e. examining the vertexes one by one, till an optimal solution is obtained. That is why it is also known as iterative method. The steps of this method is

i) First a trail basic feasible solution to the given problem is supposed.

ii) Then this trial basic feasible solution is tested for optimal solution.

iii) The first trial solution is improved by a set of rules, and repeating the process till an an optimal solution is obtained.

Since the number of extreme points of feasible solution space is finite, the value of the objective function is assumed to be improved in each iteration, i.e. from one iteration to another, and achieve optimal solution in a finite number of steps and also indicates when an unbounded solution is reached.

For convenience, re-state the LP problem in the standard form :

Optimize (Max. or Min.) $Z = c_1x_1 + c_2x_2 + ... + c_nx_n + 0s_1 + 0s_2 + ...+ 0\ s_n$

Subject to the constraints

$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n + s_1(\leq, =, \geq)\ b_1$

$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n + s_2(\leq, =, \geq)\ b_2$

....

$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n + s_m\ (\leq, =, \geq)\ b_m$

and $x_1, x_2, ..., x_n , s_1 , s_{2, ..., } s_m \geq 0$

Where *(s) denotes the slack variable, *(- s) denotes the surplus variable.

In matrix notations the above standard form can be stated as:

Optimize (Max. or Min.) $Z = cx + 0s$

Subject to the constraints

$Ax + s = b$

and $x, s \geq 0$

Where $c = (c_1, c_2, ..., c_n)$ is the row vector, $x = (x_{1,} x_{2,} ....., x_n)^T$, $b = (b_{1,} b_{2,} ....., b_m)^T$and

$s = (s_{1,} s_{2,} ....., s_m)$ are column vectors.

and $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a\ \ 1n \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ is the m x n matrix of coefficients of variables

$x_1, x_2, ..., x_n$ in the constraints.

* ***Slack variable:*** The non-negative variable which is added to the left hand side of the constraint to convert it into equation is known as a slack variable. They are added to the constraints with ≤ sign. These variables are added to the original objective function with zero coefficients.These may represents the unused resource, like labour hours, money, warehouse, time on a machine etc.

* ***Surplus variable:*** The non-negative variable which is subtracted from the left hand side of the constraint to convert it into equation is known as a surplus variable. They are subtracted to the constraints with ≥ sign. It represents amount by which solution values exceed a resource. Like slack variables, these variables are also added to the original objective function with zero coefficients.

4.8 The Simplex Algorithm(Maximization Problem):

The steps of simplex method to obtain an optimal solution, if exists, to a linear programming problem are –

Step 1. Formulation of the Mathematical model of the given problem.

To formulate the mathematical model,

i) Convert the objective function to a maximization if it is of the minimization type by the following relation:

Minimize Z = -- Maximization (- Z)

ii) All bi values must be positive. If any value is negative, then multiply the corresponding constraint by (-1) so that all bi ≥ 0 and consequently change the ≤ type constraint into a ≥ type constraint and vice versa.

iii) In the mathematical model, add the additional variables to the left side of each constraint and assign a zero cost coefficient to these in the objective function.

iv) Replace each unrestricted variable with the difference of two non-negative variables, replace each negative variable with a newnon-negative variable whose value is the negative of the original variable.

Step 2. Set up the initial solution.

To get an initial basic feasible to the given LP problem, write down the coefficients of all variables in the linear programming model in the tabular form as shown in the following :

		$c_j \rightarrow$	c_1	c_2	.....	c_n	0	0	.....	0
Coefficient of Basic Variables (C_B)	Variables in Basis B	Value of Basic Variables B($=x_B$)	Variables							
			x_1	x_2	... ..	x_n	s_1	s_2	.... .	s_n
C_{B1}	s_1	$x_{B1} = b_1$	a_{11}	a_{12}		a_{1n}	1	0		0
C_{B2}	s_2	$x_{B2} = b_2$	a_{21}	a_{22}		a_{2n}	0	0		0
. . .	. . .	. . .	. . .	. . .	.... .	. . .	. . .	. . .	. . .	. . .
C_{Bm}	s_n	$x_{Bm} = b_m$	a_{m1}	a_{m2}	... ..	a_{mn}	0	0	.... .	1
$Z = \sum C_{Bi}x_{Bi}$ = (B. V. coefficients) x (Values of B.V.)	$z_j = \sum C_{Bi}x_{Bi}$ = $\sum$ (B. V. coefficients) x (j^{th} column of data matrix.)		0	0	... ..	0	0	0	.... .	0
	$c_j - z_j \rightarrow$		$c_1 - z_1$	$c_2 - z_2$	... ..	$c_n - z_n$	0	0	.... .	0

The identity matrix in the above table is called a basis matrix (because the basic feasible solution is represented by B= I). Assign the values of b_i's to the column variables in the identity matrix.

The variables corresponding to the columns of the identity matrix are called **basic variables**and the remaining are the non-basic variables. In general, an LP model with n variables and m constraints have **m** basic variables and **(n-m)** non-basic variables.

Step 3: Test for optimality

In this step, examine the values of $(c_j - z_j)$ obtained in the above table. There may be three cases. They are—

i) All $(c_j - z_j) \leq 0$, implies that the basic feasible solution is optimal.

ii) If at least one column of the coefficients matrix (i.e.a_k) for which $(c_k - z_k) > 0$ and all elements are negative (i.e. $a_{ik} < 0$), then there exists an unbounded solution to the given problem.

iii) If at least one $(c_j - z_j) > 0$ and each of these has at least one positive element (a_{ij}) for some row, then it indicates that an improvement in the value of objective function Z is possible.

Step 4:Select the variable to enter in the basis

If in step 3, case (iii) holds, then one variable, that has the largest $(c_j - z_j)$ value, to be select to enter into the new solution. That is, $c_k - z_k = \text{Max}\ \{(c_j - z_j);\ c_j - z_j > 0\}$

Step 5:Select the variable to leave the basis

In this step,from the existing set of basic variables one variable, that have to leave the basis, is to be determined. For this each number in x_B – column (i.e. b_i values) is divided by the corresponding (positive) number in the key column and select the row for which this ratio, is non-negative and minimum. The ratio is known as ***replacement ratio*** and is defined as

$$\frac{x_{Br}}{a_{rj}} = Min\left\{\frac{x_{Bi}}{a_{rj}}\ ;\ a_{rj} > 0\right\}$$

The row selected in this way is known as ***key or pivot row*** and represents the variable that have to leave the basic solution. The element that lies at the intersection of the key row and key column of the simplex table is called ***key or pivot element***.

Step 6: Determining the new solution

i) If the key element is 1, then the row remains the same in the simplex table.

ii) If the key element is other than 1, then divide each element of the key row, including the elements of the x_B – column, by the key element to determine the new values of that row.

iii) Make all elements of the key column zero, other than the key element, by elementary row operations on all rows and thus obtain the new values for the rows.

The new entries in $\mathbf{c_B}$(coefficient of the basic variables) and $\mathbf{x_B}$ (value of basic variables) columns are updated in the new simplex table of the current solution.

Step 7: Repeat the procedure

Go to the step 3 and repeat the process until all entries in the $(c_j - z_j)$ row are either negative or zero.

1.9 ***Let us explain the simplex method with the help of an example***.

Example: Solve the following LP problem using simplex method:

Maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints

$$2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

and $x_1, x_2, x_3 \geq 0$

Solution :

Step 1:Rewrite the LP problem by introducing the slack variables s_1, s_2, s_3 to convert the inequality constraints to equality.

Maximize $Z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to the constraints

$$2x_1 + 3x_2 + s_1 = 8$$

$$2x_2 + 5x_3 + s_2 = 10$$

$$3x_1 + 2x_2 + 4x_3 + s_3 = 15$$

and $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Step 2: Since all b_i(RHS value of the above equalities) > 0, therefore, let us choose the basic feasible solution as

$x_1 = x_2 = x_3 = 0$; $s_1 = 8$, $s_2 = 10$, $s_3 = 15$ and Max Z = 0

Let us construct the first table considering this as the basic feasible solution as follows:

Table 1									
		$c_j \rightarrow$	3	5	4	0	0	0	Min. Ratio x_B/x_2
Profit per unit c_B	Variables in Basis B	Solution values B(= x_B)	x_1	x_2	x_3	s_1	s_2	s_3	
0	s_1	8	2	③	0	1	0	0	8/3 →
0	s_2	10	0	2	5	0	1	0	10/2
0	s_3	15	3	2	4	0	0	1	15/2
Z = 0		z_j	0	0	0	0	0	0	
		$c_j - z_j$	3	5 ↑	4	0	0	0	

Step 3: Calculate $c_j - z_j$, where z_j are calculated as—

z_j = (Basic variable coefficient, c_B) x (j^{th} column data)

Thus, for non-basic variables $x_1, x_2,$ and x_3 ,

$z_1 = 0(2) + 0(0) + 0(0) = 0$, for column x_1

$z_2 = 0(3) + 0(2) + 0(2) = 0$, for column x_2

$z_3 = 0(0) + 0(5) + 0(4) = 0$, for column x_3

Therefore,

$$c_1 - z_1 = 3 - 0 = 3$$

$$c_2 - z_2 = 5 - 0 = 5$$

$$c_3 - z_3 = 4 - 0 = 4$$

and the value of the objective function is given by—

Z = (Basic variable coefficient, c_B) x (Basic variable values, x_B)

= 0(8) + 0(10)+ 0(15) = 0

Since all $c_j - z_j \geq 0$ (j= 1,2,3), the current solution is not an optimal solution.

From **Table 1**above, variable x_2 is chosen to enter in the basis as $c_2 - z_2$is the largest positive number in the x_2 column, where all elements are positive. The x_2 column is the key column.

Step 4:In **table 1**, Min. Ratio is obtained corresponding to the row 1, therefore the corresponding basic variable $\mathbf{s_1}$is chosen to leave the solution.

Step 5: Since the key element enclosed in the circle in Table 1 is not 1, therefore divide all elements of the key row by 3 to obtain the new values of the elements in this row.

Now, construct the new Table 2 as follows:

Table 2									
		c_j→	3	5	4	0	0	0	Min. Ratio x_B/x_2
Profit per unit c_B	Variables in Basis B	Solution values B(= x_B)	x_1	x_2	**x_3**	s_1	s_2	s_3	
5	x_2	8/3	2/3	1	0	1/3	0	0	--
0	s_2	14/3	- 4/3	0	(5)	- 2/3	1	0	(14/3)/5→
0	s_3	29/3	5/3	0	4	- 2/3	0	1	(29/3)/4
Z = 40/3		z_j	10/3	5	0	5/3	0	0	
		$c_j - z_j$	-1/3	0	4 ↑	-5/3	0	0	

Since $c_3 - z_3 > 0$, therefore the current solution is not an optimal solution.

Step 6:

Let us construct the new Table with new variable (i.e. repeating the steps 3 to 5)

Table 3									
		c_j→	3	5	4	0	0	0	Min. Ratio x_B/x_2
Profit per unit c_B	Variables in Basis B	Solution values B(= x_B)	x_1	x_2	**x_3**	s_1	s_2	s_3	
5	x_2	8/3	2/3	1	0	1/3	0	0	(8/3) (2/3)
4	x_3	14/15	- 4/15	0	5	- 2/15	1/15	0	--

0	s_3	89/15	41/15	0	4	2/15	-4/5	1	(89/15(41//15)→
Z = 256/15		z_j	34/15	5	0	5/3	4/5	0	
		$c_j - z_j$	11/15 ↑	0	4	-5/3	-4/5	0	

Since, $c_1 - z_1 > 0$; therefore, still an optimalsolutionis not obtained.

Again, construct another new Table with new entering variable .

Table 4								
		c_j→	3	5	4	0	0	0
Profit per unit c_B	Variables in Basis B	Solution values B(= x_B)	x_1	x_2	**x_3**	s_1	s_2	s_3
5	x_2	50/41	0	1	0	15/41	8/41	-10/41
4	x_3	62/41	0	0	1	- 6/41	5/41	4/41
3	x_1	89/41	1	0	0	- 2/41	-12/41	15/41
Z = 765/41		z_j	3	5	4	45/41	24/41	11/41
		$c_j - z_j$	0 ↑	0	0	-45/41	-24/41	-11/41

In Table 4 above, all $c_j - z_j < 0$ for non basic variables. Therefore, optimal solution to the given LP problem is attained here and the values of the variables are:

x_1=89/41, x_2 =50/41, x_3 =62/41 and the optimal value is Z = 765/41.

Example: Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to the constraints

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 2x_3 \leq 460$$

$$x_1 + 4x_2 \leq 420$$

and $x_1, x_2, x_3 \geq 0$

Solution:

Let us introduce the slack variables s_1, s_2, s_3 to convert the inequality constraints to equality. The LP problem becomes:

Maximize $Z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to

$$x_1 + 2x_2 + x_3 + s_1 = 430$$

$$3x_1 + 2x_3 + s_2 = 460$$

$$x_1 + 4x_2 + s_3 = 420$$

and $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Since all b_i(RHS value of the above equalities) > 0, therefore, the initial simplex table is follows:

Table 1									
		$c_j \rightarrow$	3	2	5	0	0	0	Min. Ratio x_B/x_3
Profit per unit c_B	Variables in Basis B	Solution values B(= x_B)	x_1	x_2	**x_3**	s_1	s_2	s_3	
0	s_1	430	1	2	1	1	0	0	430/1
0	s_2	460	3	0	(2)	0	1	0	460/2 →
0	s_3	420	1	4	0	0	0	1	---
Z = 0		z_j	0	0	0	0	0	0	
		$c_j - z_j$	3	2	5 ↑	0	0	0	

Since all $c_j - z_j \geq 0$ (j= 1,2,3), the current solution is not an optimal solution.

From **Table 1**above, variable x_3 is chosen to enter in the basis as $c_3 - z_3$is the largest positive number which fall in the x_3 column, where all elements are positive. The x_3 column is the key column.

In **table 1**, Min. Ratio is obtained corresponding to the row 2, therefore the corresponding basic variable **s_2**is chosen to leave the solution.

Now, construct the new improved simplex table (Table 2) as follows:

Table 2									
		$c_j \rightarrow$	3	2	5	0	0	0	Min. Ratio
Profit	Variables	Solution	x_1	x_2	**x_3**	s_1	s_2	s_3	

per unit c_B	in Basis B	values B(= x_B)							x_B/x_2
0	s_1	200	-1/2	②	0	1	-1/2	0	(200/2)→
5	x_3	230	3/2	0	1	0	1/2	0	---
0	s_3	420	1	4	0	0	0	1	(420/4)
Z = 1150		z_j	15/2	0	5	0	5/2	0	
		c_j- z_j	-9/2	2 ↑	0	0	-5/2	0	

Since c_2- z_2>0, therefore the current solution is not an optimal solution.

Now, construct the new improved simplex table (Table 3) as follows:

Table 3								
		c_j→	3	2	5	0	0	0
Profit per unit c_B	Variables in Basis B	Solution values B(= x_B)	x_1	x_2	**x_3**	s_1	s_2	s_3
2	x_2	100	-1/4	1	0	1/2	-1/4	0
5	x_3	230	3/2	0	1	0	1/2	0
0	s_3	20	2	0	0	-2	1	1
Z = 1350		z_j	7	2	5	1	2	0
		c_j- z_j	-4	0	0	-1	-2	0

In Table 3 above, all c_j- z_j< 0 for non basic variables. Therefore, optimal solution to the given LP problem is attained here and the values of the variables are:

x_1=0, x_2 =100, x_3 =230 and the optimal value is Z = 1350.

4.10Artificial Variable Technique

INTRODUTION

The LPP with constraints having ≥ and = signs and all $b_i \geq$, the basis of matrix cannot be obtained as an identity matrix in the starting simplex table.Therefore a new type of variable called the artificial variable is introduced. The only purpose of introducing these variables is to get the

starting basic feasible solution, so that simplex procedure may be adopted as usual until the optimal solution is obtained

To solve such types of LPP, there are two methods.

1. The Big *M* Method or Method of Penalties.
2. The Two-phase Simplex Method.

Here we will discuss only the Big M method.

Big M method

The Big-M-Method is a method, through which we can remove the artificial variables from the basis. To solve a L.P.P by simplex method, we have to start with the initial basic feasible solution and construct the initial simplex table. However, in some problems where atleast one of the constraint is of = or ≥ type., the slack variables cannot provide the initial basic feasible solution. The Big-M-Method is used to solve such types of L.P.P.

If the objective function Z is to be minimized(maximized), then a very large positive (negative) price, called penalty, is assigned to each artificial variable. The steps of the method are:

Step 1: Express the LPP in standard form by adding the slack, surplus and artificial variables if any. Assign a zero coefficient to both slack and surplus variables and a very large positive coefficient +M (minimization case) and –M (maximization case)to artificial variable in the objective function.

Step 2: The initial basic feasible solution is obtained by assigning zero value to original variables.

Step 3: Calculate the values of $c_j - z_j$ in last row of the simplex table and examine the following:

i) If all $c_j - z_j \geq 0$, then the current basic feasible solution is optimal.

ii) If for a column, k, $c_k - z_k$ is most negative and all entries in this column are negative, then the LPP has an unbounded optimal solution.

iii) If one or more $c_j - z_j < 0$ (minimization case) , then select that variable to enter into the basis with the largest negative $c_j - z_j$ value.

Step 4: Proceed similarly as in the simplex method.

Example: Solve the following LPP

Minimize $Z = 5x_1 + 3x_2$

Subject to

$2x_1 + 4x_2 \leq 12$

$2x_1 + 2x_2 = 12$

$5x_1 + 2x_2 \geq 10$

and $x_1, x_2 \geq 0$

Solution:

The standard form of the LPP after introducing the slack variable s_1, surplus variable s_2, and artificial variable A_1 and A_2, is

Minimize $Z = 5x_1 + 3x_2 + 0s_1 + 0s_2 + MA_1 + MA_2$

Subject to

$2x_1 + 4x_2 + s1 = 12$

$2x_1 + 2x_2 + A1 = 12$

$5x_1 + 2x_2 - s2 + A2 = 10$

and $x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$

The initial solution table is—

Table 1

		$c_j \rightarrow$	5	3	0	0	M	M	Min. Ratio
Profit per unit c_B	Variables in Basis B	Solution values B(= x_B)	x_1	x_2	**s_1**	s_1	A_1	A_2	x_B/x_1
0	s_1	12	2	4	1	0	0	0	12/ 2= 6
M	A_1	10	2	2	0	0	I	0	10/2 = 5
M	A_2	10	(5)	2	0	-1	0	1	10/5 = 2 → Outgoing variable
Z = 20M		z_j	7M	4M	0	- M	M	M	
		$c_j - z_j$	5-7M ↑ Entering variable	3 – 4M	0	M	0	0	

As the key element is 5 (circled in the table above), make it 1 and other elements of the column zero. Introducing the variable x_1 and removing A_2 from the basis, the new improved solution table is constructed as follows:

Table 2

		$c_j \rightarrow$	5	3	0	0	M	Min. Ratio
Profit per unit c_B	Variables in Basis B	Solution values $B(= x_B)$	x_1	x_2	$\mathbf{s_1}$	s_2	A_1	x_B/x_1
0	s_1	8	0	(16/5)	1	2/5	0	8/ (16/5) = 5/2 → Outgoing variable
M	A_1	6	0	6/5	0	2/5	I	6/ (6/5) = 5
5	x_1	2	1	2/5	0	-1/5	0	2/ (2/5) = 5
Z = 10+6M		z_j	5	(6M/5)+2	0	(2M/5)-1	M	
		$c_j - z_j$	0	(-6M/5)+1 ↑	0	(-2M/5)+1	0	

Entering variable

Introducing x_2 and removing s_1 from basis, the new improved solution table(Table 3) is

Table 3

		$c_j \rightarrow$	5	3	0	0	M	Min. Ratio
Profit per unit c_B	Variables in Basis B	Solution values $B(= x_B)$	x_1	x_2	$\mathbf{s_1}$	s_2	A_1	x_B/x_1
3	x_2	5/2	0	1	5/16	1/8	0	(5/2)/ (1/8) = 40
M	A_1	3	0	0	-3/8	(1/4)	I	3/ (1/4) = 12 → Outgoing variable
5	x_1	1	1	0	-1/8	-1/4	0	---
Z = 25/2+3M		z_j	5	3	(-3M/8)+ 5/16	(M/4)-7/8	M	
		$c_j - z_j$	0	0	0	(-M/4)+7/8 ↑	0	

Entering variable

Introducing s_2 and removing A_1 from basis, the new improved solution table(Table 4) is

Table 4						
		$c_j \rightarrow$	5	3	0	0
Profit per unit c_B	Variables in Basis B	Solution values B(= x_B)	x_1	x_2	$\mathbf{s_1}$	s_2
3	x_2	1	0	1	1/2	0
0	s_2	12	0	0	-3/2	1
5	x_1	4	1	0	-1/2	0
Z = 23		z_j	5	3	-1	0
		$c_j - z_j$	0	0	1	0

Since all $c_j - z_j \geq 0$ in table 4, the optimal solution is attained here. The solution is $x_1 = 4$, $x_2 = 1$, $s_1 = 0$, and $s_2 = 12$ and the minimum value of Z = 23.

Check Your Progress:

1. For maximization LP problem, the simplex method is terminated when all values

i) $c_j - z_j \leq 0$, ii) $c_j - z_j \geq 0$, iii) $c_j - z_j = 0$, iv) $z_j \leq 0$

2. If for a given solution, a slack variable is equal to zero, then

i) The solution is optimal. Ii) The solution is infeasible,

iii) The entire amount of resource with the constraint in which the slack variable appears has been consumed.

3. If an optimal solution is degenerate, then

i) the solution is infeasible ii) the solution is of no use to the decision maker,

iii) None of the above.

4. If a negative value appears in the solution values (x_B) column of the simplex table

i) the solution is optimal, ii) the solution is infeasible, iii) the solution is unbounded

5. For maximization problem, the objective function coefficient for an artificial variable is

i) + M ii) – M iii) Zero iv) None of the above.

Check Your Progress:

Solve the following LPP using Simplex method:

1. Maximize $Z = x_1 + x_2 + x_3$

   Subject to

   $4x_1 + 5x_2 + 3x_3 \leq 15$

   $10x_1 + 7x_2 + x_3 \leq 12$

   and $x_1, x_2, x_3 \geq 0$

Ans: $x_1 = 0$, $x_2 = 0$, $x_3 = 5$ and Max $Z = 5$

2. Minimize $Z = 5x_1 + 8x_2$

   Subject to the constraints

   $3x_1 + 2x_2 \geq 12$

   $x_1 + 4x_2 \geq 4$

   $x_1 + x_2 \leq 5$

   and $x_1, x_2 \geq 0$

Answer: $x_1 = 0$, $x_2 = 5$, and Min $Z = 40$

3. Maximize $Z = x_1 + x_2 + x_3$

   Subject to

   $x_1 + x_2 + x_3 + x_4 = 4$

   $x_1 + 2x_2 + x_3 + x_4 = 4$

   $x_1 + 2x_2 + x_3 = 4$

Ans: $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ and Max $Z = 4$

References:

1. Swarup K., Gupta P.K., Mohan M.(1990), *Operations Research*, Sultan Chand & Sons.
2. Sharma J.K. (2003), *Operations Research Theory and Applications*, MACMILLAN.
3. Gupta P. K., Hira D.S.(2014), *Operations Research*, S. Chand & Company Pvt. Ltd.
4. Sharma S.D.(2002), *Introduction To Operations Research*, Kedar Nath Ram Nath & CO.

Block 2 : Unit 1

Assignment Problem :

Unit Structure:

1.1 Introduction

Assignment problem is a special type of linear programming problem, which is particularly useful in a situation where a certain number of tasks or jobs is required to be assigned to equal number of facilities or persons, one job to each person at a minimum cost. The main objective of the assignment problem is to minimize the total assignment cost or to minimize the total consumed time for executive of all the jobs. Here we make the assumption that each person can perform each job, but with carrying degree of efficiency.

Real life situations where assignment technique can be successfully used are :

- sales persons to sales counters.
- vehicles to routes.
- products to factors
- project engineers to projects.

1.2 Objectives

After studying this unit you will be able to,

(i) understand the features of 'Assignment Problem' and how it is used in Industry and Business.
(ii) how to formulate an assignment problem.
(iii) Methods of solving Assignment problems– use of Hungarien method.
(iv) Understand what is a Transportation problem and how it is used in Industry and Business.
(v) formulate a transportation problem.

1.3 Example of an Assignment Problem :

Suppose there are n jobs in a factory and the factory has n machines to perform this jobs. Suppose, a job, ra, i (i=1, 2, ..., n) when performed by the machine j (j=1, 2, ..., n) in curns a cost of C_{ij}. The assignment is to be made in such a way that each job can be associated with one and only one machine. Determine an assignment of job to machines so as to minimize the overall cost.

1.4 Mathematical Formulation of Assignment Problem

Let us consider a variable x_{ij} (i=1, 2, ..., n, j=1, 2, ..., n), as given below :

$$x_{ij} = \begin{cases} 0, \text{if the } i^{th} \text{ person is not assigned to the job j} \\ 1, \text{if , the } i^{th} \text{ person is assigned to the job j} \end{cases}$$

since one job is to be assigned to only one machine,

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad \sum_{j=1}^{n} x_{ij} = 1$$

Suppose, c_{ij} = cost required for machine j to perform the job i.

total assignment cost is, $z = \sum_{j=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

Thus, mathematically we can define an assignment problem as,

$$\text{minimize } z = \sum_{c=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to the constraints,

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j=1, 2, ..., n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i=1, 2, ..., n$$

with x_{ij}=0 or 1 for all i and j.

1.5 Important theorem

Theorem 1 : If in an assignment problem a constant is added or subtracted from every element of a row (column) of the cost matrix $[c_{ij}]$, these an assignment which minimizes total cost for one matrix also minimizes the total cost of the other matrix. $z' = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

Mathematically, if, $x_{ij} = \hat{x}_{ij}$ minimizes

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \text{ such that}$$

$$\sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{n} = 1 \text{ and } x_{ij} \geq 0,$$

then $x_{ij} = \hat{x}_{ij}$ also minimizes $x' = \sum_{i=1}^{n} \sum_{j=1}^{n} c'_{ij} x_{ij}$,

where, $c'_{ij} = c_{ij} \pm u_i \pm v_j$

u_i and v_j are constants, i=1, 2, ..., n, j=1, 2, ..., n

Proof :

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \left(c_{ij} \pm u_i \pm v_1 \right) t_{ij}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \pm \sum_{i=1}^{n} u_i \sum_{j=1}^{n} x_{ij} \pm \sum_{j=1}^{n} u_j \sum_{i=1}^{n} t_{ij}$$

$$= z \pm \sum_{i=1}^{n} u_i + \sum_{j=1}^{n} v_j \because \sum_{i=1}^{n} t_{ij} = 1 = \sum_{j=1}^{n} t_{ij}$$

Now, since $\sum_{i=1}^{n} u_i$ and $\sum_{j=1}^{n} v_j$ are independent of x_{ij}, z' is minimized when z is minimized.

Hence, , which minimizes z also minimizes z'.

Self Assessment Problem

1. Define Assignment problem. Give one application of assignment problem in health care administration.
2. Give the mathematical formulation of an assignment problem.
3. State and prove the Reduction theorem.

1.6 Solution of Assignment Problem :

Small size assignment problems can be solved by evaluating all combinations and selecting the optimal one. However, this process becomes very time consuming as the magnitude of the problem gets bigger. To overcome then difficulties, an efficient method to value assignment problems was developed by Prof. D. Konig, a Hungarien Mathematician, known as Hungarian method of Assignment Problem or Reduced Matrix method. This method is band on the concept of opportunity cost. Opportunity cost may be regarded as the relative penalty associated with assignment of resources to activity as against making the best cost assignments. By reducing the cost matrix with atleast one zero in each now and each column, the optimal assignment is possible $\hat{x}_{ij} = x_{ij}$

Hungarien Method or Reduced Matrix Method of solving an Assignment Problem :

Hungarien method is the most efficient method for solving an assignment problem that ..... minimization objective. The procedural steps to solve a standard assignment problem are as follows.

Step 1 : Express the problem into a n n assignable cost matrix.

Step 2 : Subtract the minimum element of each row in the cost matrix from every element of the corresponding row.

Step 3 : Now select the smallest element of each column of the reduced matrix from step 2 and subtract it from every element of that column. The resulting matrix is the starting effectiveness matrix.

Step 4 : Examine the rows one by one, starting with 1st row of the matrix obtained in step 2. Until a row with exactly one zero is found. Mark this zero by exclosing it in a square (□), indicating that an assignments will be made there. Cross out ( ) all other zeros in the column (in which □ appears), showing that the cells marked will not be considered for any further assignments, continue in this manner until all the rows are era.

(b) After examining all the rows, examine all the columns, until a column with single unmarked zero is found. Mark □ this zero and cross out all other zeros in the row in which □ is marked. Proceed in this way until all the columns are taken care of.

(c) Continue the operations (a) and (b) necessarily till one of the following occurs.

(i) there is no row and no column without assignment. In such care, complete optimal solution is obtained.

(ii) There may be some row or column without an assignment. In such a case the current solution is not optimal and we have to modify the cost matrix by adding or subtracting to create some more zero in it as explained in step 5.

Step 5 : When the matrix obtained in step 4 doesnot contain assignment in every row and in every column, we draw minimum number of horizontal and vertical lineer, so that all zeros are covered at least once. For this we proceed as follows :

(i) Mark (3) all rows for which assignments havenot been made.

(ii) Mark (3) all columns which have zeros in marked row.

(iii) Mark (3) rows (not already marked) which have assignments in marked columns.

(iv) Repeat steps (ii) and (iii) until no more marking is possible.

(v) Draw minimum number of lines through unmarked rows and through unmarked columns. Let the number of lines be N.

If (i) N=n, the current assignment is optimal,

(ii) N<n, the current assignment is not optimal and move to step 6.

Step 6 : Examine all the elements of the cost matrix and select the smallest uncovered element, that is an element that do not have a line through them. Add this smallest element to every element that lies in the intersection of two lines, and subtract that smallest element from all, the elements that donot have a line through them. The resulting matrix is the new effectiveness matrix.

Step 7 : Go to step 4 and repeat the steps until an optimal solution is obtained.

Self Assessment Problem

1. Explain the Hungarian method to solve an assignment problem.

Illustrated Examples :

1. A company produce four products A, B, C, D, which can be manufactured in each of the four machines I, II, III, IV. The processing cost of each product in each machine is given below :

Machine → / Products ↓	I	II	III	IV
A	12	30	21	15
B	18	33	9	31
C	44	25	24	21
D	23	30	28	14

To achieve maximum processing cost, which product will you process in which machine.

Sol^{n} : Step 1 : Subtracting the smallest element of each row from every element of the corresponding row, we get the following matrix :

Machine → / Products ↓	I	II	III	IV
A	0	18	9	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

Step 2 : Subtracting smallest element of each column from every other element of the corresponding column, we get the following matrix :

Machine / Products	I	II	III	IV
A	0	14	9	3
B	9	20	0	22
C	23	0	3	0
D	9	12	14	0

Step 3 : Next we start to make assignments using zeros. Starting with row 1 we mark □, i.e make assignments in the row containing only one zero and cross out (×) the zeros in the column where □ appears. Accordingly we get the following table.

	I	II	III	IV
A	[0]	14	9	3
B	9	20	[0]	22
C	23	[0]	3	~~0~~
D	9	12	14	[0]

Step 4 : Again we make assignments starting with column 1 by putting □ around any unmarked or uncrossed single zero in any column of the above table. Here also we cross out any other zeros in the same row where □ is marked. Accordingly we get the following table :

	I	II	III	IV
A	[0]	14	9	3
B	9	20	[0]	22
C	23	[0]	3	~~0~~
D	9	12	14	[0]

From the above table we see that every row and every column have an assignment. So we have arrived at an optimal solution as follows :
Product A will be assigned to machine I
Product B will be assigned to machine III
Product C will be assigned to machine II
Product D will be assigned to machine IV

Example 2 :

A computer centre has three expect programmers. The centre wants three application programmes to be developed. The head of the computer centre after studying carefully the programmes to be developed, estimates the computation time in minutes required by the experts for the application programmes as follows :

		Programmers		
		A	B	C
Programmes	1	120	100	80
	2	80	90	110
	3	110	140	120

Assign the programmers to the programmes in such a way that the total computer time is minimum.

Sol^{n} :

Step 1 : Subtracting the smallest element of each row from every element of the corresponding row, we get the following matrix.

	A	B	C
I	40	20	0
II	0	10	30
III	0	30	10

Step 2 : Subtracting the smallest element of each column from every element of the corresponding column we get,

	A	B	C
I	40	10	0
II	0	0	30
III	0	20	10

Step 3 : Now we test whether it is possible to make assignments using the zeros. Starting with row 1, we mark □, i.e. make assignments in the row containing only one zero and cross any other zeros in the corresponding column where □ lies.

	A	B	C
I	40	10	□ 0
II	~~0~~	0	30
III	□ 0	20	10

Step 4 : Again starting with column 1, mark □ in the column containing only one unmarked or uncrossed zero in the above table and cross out the zeros in the corresponding row in which □ is marked.

	A	B	C
I	40	10	□ 0
II	~~0~~	□ 0	30
III	□ 0	20	10

Now since the number of assignments is equal to the number of rows, the optimal solution is obtained. Hence, the pattern of assignment of programmers and programmes along with their respective time is given below :

Programmer	Programme	Time (in mins)
1	C	80
2	B	90
3	A	110

Exercise 3 : A departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and tasks differ in their intrinsic difficulty. His estimate of the times each man would take to perform each task is given in the effectiveness matrix below. How should the task be allocated, one to a mass, so as to minimize the total man hour.

Tasks \ Subordinates	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

Sol^{n}. Step 1 : Subtracting the smallest element of each row from every element of the corresponding row, we get the gollowing matrix.

	I	II	III	IV
A	0	18	11	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

Step 2 : Substracting the smallest element of each column from every element of the corresponding column, we get the following matrix.

	I	II	III	IV
A	0	14	11	3
B	9	20	0	22
C	23	0	3	0
D	9	12	14	0

Step 5 : Now we make the assignments using the zeros. Starting with row 1 we mark □, i.e make assignments to the single zero in the row and cross (×) the zeros in the corresponding column in which □ lies. Accordingly we get the following matrix.

	I	II	III	IV
A	□ 0	14	9	3
B	9	20	□ 0	22
C	23	0	3	× 0
D	9	12	14	□ 0

Again starting with column 1, we mark □ in the column containing only one unmarked or uncrossed zero in the above table and cross out any other zero in the corresponding row in which □ is marked. So the resultant matrix is,

	I	II	III	IV
A	□ 0	14	9	3
B	9	20	□ 0	22
C	23	□ 0	3	× 0
D	9	12	14	□ 0

Since in the above table every row and every column has an assignment, the following optimum assignment schedule is complete. Hence for maximum man hour the allotment should be as follows :

Tasks : A B C D

Subordinates :	I	II	III	IV
Man hours :	8	4	19	10

Hence the minimum man hour required is

8+4+19+10= 41 hrs.

Example 4 : A company has six jobs to be processed by six machines. The following table gives the return in rupees when the i^{th} job is assigned to the j^{th} mechanic. (i, j = 1, 2, ..., 6). How should the jobs be assigned to themechanics so as to maximize the overall return?

		Job					
		I	II	III	IV	V	VI
Mechanic	A	9	22	58	11	19	27
	B	43	78	72	50	63	48
	C	41	28	91	37	45	33
	D	74	42	27	49	39	32
	E	36	11	57	22	25	18
	F	3	56	53	31	17	28

Sol^{n} : Step 1 : Subtracting the smallest element of each row from every other element of the corresponding row we get the following matrix.

	I	II	III	IV	V	VI
A	0	13	49	2	10	18
B	0	35	29	7	20	5
C	13	0	63	9	17	5
D	47	15	0	22	12	5
E	25	0	46	11	14	7
F	0	53	50	28	14	25

Step 2 : Subtracting the smallest element of each column from every element of the corresponding column use get the following reduced matrix.

	I	II	III	IV	V	VI
A	0	13	49	0	0	13
B	0	35	29	5	10	0
C	13	0	63	7	7	0
D	47	15	0	20	2	0
E	25	0	46	9	4	2
F	0	53	50	26	4	20

Step 3 : Next we make zero assignments to rows and columns releatedly until all zeros are either market □ or crossed. Accordingly we get the following matrix.

	I	II	III	IV	V	VI
A	~~0~~	13	49	[0]	~~0~~	13
B	~~0~~	35	29	5	10	[0]
C	13	~~0~~	63	7	7	~~0~~
D	47	15	[0]	20	2	~~0~~
E	25	[0]	46	9	4	2
F	[0]	53	50	26	4	20

Since row 3 and column 5 have no assignments, we proceed as follows :
Step 4 : In this step we draw minimum number of lines to cover all zeros atleast once. For this we proceed as follows :

	I	II	III	IV	V	VI	
A	~~0~~	13	49	[0]	~~0~~	13	
B	~~0~~	35	29	5	10	[0]	3 (5)
C	13	~~0~~	63	7	7	~~0~~	3 (1)
D	47	15	[0]	20	2	~~0~~	
E	25	[0]	46	9	4	2	3 (4)
F	[0]	53	50	26	4	20	3 (7)
	3	3				3	
	(0)	(2)				(3)	

(1) we mark (3) in the row 3 where there is no assignment.
(2) Then we mark (3) columns 2 and 6, where there are zeros in the marked row 3.
(3) Then we mark rows 5 and 2 which have assignments in the marked columns 2 and 6.
(4) Then we mark column 1 (not already marked) which have zero in the marked row 2.
(5) There we mark row 6, which have assignment in the marked column 1.
(6) There we draw liners through all marked columns 1, 2, 6 and through unmarked rows 1 and 4.
Step 5 : Now from the above table, check the smallest element which donot have a live through it. It is 4. Now subtract this element 4 from all the elements that donot have a line through them and add to all elements that lies at the intersection of two lines and leaving remaining elements unchanged get the following matrix.

	I	II	III	IV	V	VI
A	4	17	49	0	0	17
B	0	35	25	1	6	0
C	13	0	59	3	3	0
D	51	19	0	20	2	4
E	25	0	42	5	0	2
F	0	53	46	22	0	20

Step 6 : We repeat the step 3 and mark 0 in row 3 and in column 4 and cross zero in 1st row. Now since all other rows and columns have more then one zero. We arbitrarily mark zero in (B, I) position and cross zero in (B, VI) and (F, I) position. Then mark assignment in (C, VI) position and cross in (C, II) position. Continuing like this we make zero assignments and get the following reduced matrix.

	I	II	III	IV	V	VI
A	4	17	49	[0]	~~0~~	17
B	[0]	35	25	1	6	~~0~~
C	13	~~0~~	59	3	3	[0]
D	51	19	[0]	20	2	4 →
E	25	[0]	42	5	~~0~~	2
F	~~0~~	53	46	22	[0]	20

Now since all the rows and columns have assignments optimal solution is attained. Hence the optimal assignment is,

A IV, B I, C VI, D III, E II, F V

Check your progress

1. A job productrion unit has four jobs A, B, C, D which can be manufactured in each of the four machines P, Q, R and S. The processing cost of each job on each machine is given in the table below:

Jobs	Processing cost of machines			
	P	Q	R	S
A	31	25	33	29
B	25	24	23	21
C	19	21	23	24
D	38	36	34	40

To achieves minimum processing cost, which job will you process on which machine.

2. A department has five employees with five jobs to be performed The time (in four) each man will take to perform each job is given in the following matrix :

		Job				
		I	II	III	IV	V
Jobs	A	9	22	58	11	19
	B	43	78	72	50	63
	C	41	28	91	37	45
	D	74	42	27	49	39
	E	36	11	57	22	25
	F	3	56	53	31	17

How should the jobs be allocated, one employee, ao as to minimize the total man hours?

Transportation Problem :

Introduction : The transportation problem is a particular class of linear programming problem entended to establish 'low cost route' of tranportation of various quantities of a single homogeneous commdity. Thar are initially stored at different origins to different destinations. Transportation problems can easily be expressed as linear programming problems, which can be solued by the simplex method. However, due to the presence of a large number of variables and constrants, it becomes very time consuming to solve such a problem. To overcome this difficulty, transportation algorithms, namely, the stepping store method, Modified Distribution Method have been developed for this purpose.

The transportation algorithmn that will he discussed in this chapter is applied to minimization of total cost of transportation of a single homogeneous quantity. For example, suppose a company producing tyres has two factories in different locations and there sales partner are three central warehouses. The factories can produce a given number of tyres per week and the expected demand of the warehouses are also known. There is a difenite shipping cost from each factory to each warehouse. Our transportation problem will be to determine which factory should produce and ship how many tyres to which warehouse to meet the demand with minimum cost.

Mathematical Formulation of Transportation Problem

Suppose a company has 'm' plants located in m different places manufacturing an identical product. The units produced atthese 'm' plants are required to be shipped to the 'n' warehouses located in 'n' different places. The units can be shipped in different ways to the warehouse with different amounts of cost. The objective of the transportation problem will be, therefore to find out the route that entails minimum cost. Suppose that m plant locations (origins) are $O_1, O_2, ..., O_m$ and n warehouses (destinations) are $D_1, D_2, ..., D_n$. Suppose that m plant produce non-negative quantity a_i=(i=1, 2, ...m) of a product and non negative quantity b_j (j=1, 2, ..., n) of the same product are required at the n warehouses, such that the

total quantity produced is equal to the total quantity required.

—— (1)

Also let C_{ij} be the cost of transportation of a unit from i^{th} origin to j^{th} destination. Then the problem is to find x_{ij}, the quantity transported from i^{th} origin to j^{th} destination in such a way that the total transportation cost

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \text{ is minimized}$$

i.e. minimize, $z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ —— (2)

Subject to the constraints :

$$\left.\begin{aligned}\sum_{j=1}^{n} x_{ij} &= a_i, \quad i = 1,2,.....,m \\ \sum_{i=1}^{n} x_{ij} &= b_j, \quad j = 1,2,.....,n\end{aligned}\right\} \text{—— (2)}$$

and $x_{ij} \geq 0 \; \forall$ i and j

Thus we see that the transportation problem is an L.P.P. of special type, where we are required to find the values of m, n variables that minimize the objective for z, given by (2), satisfying (m+n) restriction as given in (3), restriction (1) and non-negative restrictions of the variables.

LQ $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Check Your Progress

1. Define a transportation problem with suitable example.
2. Explain transportation problem and show how it can be considered as an L.P.P.
3. Give the mathematical formulation of a transportation problem.

1.7 Existance of a Feasible Solution :

Theorem– a necessary and sufficient condition for the existance of a feasible solution to a transportation problem given by (2) is that,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Proof : Necessary condition :

Let us assume that there exists a feasible solution to the transportation problem (2). Obviously we have,

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i$$

and $$\sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} b_j$$

From the above two equations we get,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Hence the condition is necessary.

Sufficient Condition : Here we assume that,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

then there exists a feasible solution to the transportation problem. Let it be,

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = k.(\text{say})$$

If $x_{ij} = \lambda_i b_j$ for all i=1, 2, ..., m ij=1, 2, ..., n where $\lambda_i \neq 0$ is any real number, then λ_i is given by,

$$\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} \lambda_i b_j = \lambda_i \sum_{j=1}^{n} b_j = \lambda_i k$$

$$\text{or } \lambda_i = \frac{1}{k} \sum_{j=1}^{n} x_{ij}$$

$$= \frac{a_i}{k}$$

$\therefore$ $$x_{ij} = \lambda_i b_j = \frac{a_i b_j}{k} \geq 0$$ for all i and j.

Hence, a feasible solution to the transportation problem exists.

1.8 Basic Feasible solution of a Transportation Problem

As we have seen that the transportation problem is a special care of an L.P.P., a basic feasible solution of a transportation problem has the same determinitions as an L.P.P., which is already discussed. However, due to redundancy in the constraints of the transportation problem, in the care of a transportation problem there are only m+n–1 basic variables out of m × n unknown variables.

1.9 The Transportation Table :

As discussed earlier, the transportation problem is just a special care of LPP and hence simplex method would obviously give an optimal solution of the problem. However it is for more easy to solve this problem by transportation method. For this the given transpor-

tation problem have to be expressed in a tabular form, known as transportation table. A specimen of the transportation table for 3 origins and 4 destinations is given below :

Distinction / Origin	D_1	D_2	D_3	D_4	Availability
O_1	C_{11}	C_{12}	C_{13}	C_{14}	a_1
O_2	C_{21}	C_{22}	C_{23}	C_{24}	a_2
O_3	C_{31}	C_{32}	C_{33}	C_{34}	a_3
Requirement	b_1	b_2	b_3	b_4	$= \sum_{j=1}^{n} b_j$

Here 3 4=12 large squares are called cells. The per unit cost c_{ij} of transportation from i^{th} origin to j^{th} destination is displayed in the lower right position of the $(i, j)^{th}$ cell. Any feasible solution to the T.P. is displayed in the table by entering the value of x_{ij} in the small square, in the upper left position of the $(i, j)^{th}$ cell. The various origin capacities and distination requirements are listed in the right most column and bottom row respectively. The feasibility of the solution can be varified by summing x_{ij} across rows and down the column.

Check your progress

1. Prove that a transportation problem always possesses a feasible solution.
2. If a transportation problem have 4 factories and 5 retail shops. What is the number of variables and the number of constraints?

Note :

1. When the total supply equals total demand, the problem is called a Balanced Transportation problem, otherwise the transportation problem is unbalanced. An unbalanced transportation problem can be made balanced by adding a dummy demand centre or dummy supply centre.
2. Where the number of positive allocations at any stage of the feasible solution is less than the number of independent constraints, the solution is said to be degenerate, otherwise non-degenerate.

1.10 References:

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Block 2 : Unit 2

To Find the Initial Basic Feasible Salutation, to Derive Optimal Solution by Modi Method, Difference Between a Transportation Problem and an Assignment Problem

Unit Structure:

2.1 Introduction

In the precious chapter we have discussed clearly about a transportation problem and also how to construct a transportation table. Now to solve a transportation problem, following algorithm may be followed.

Step 1 : Formulate the problem and construct a transportation table entering the origin capaciteies a_i, destinations requirements b_j of transportation of a unit from i^{th} origin to j^{th} destination, (i=1, 2, ...m, i=1, λ=n) as explained in unit 1.

Step 2 : Obtain an initial basic feasible solution. In this chapter we shall discuss three different methods to obtain an initial basic feasible solution. They are–

(i) North-West corner method

(ii) Least cost method

(iii) Vogel's Approximation method.

The initial solution thus obtained must satisfy the conditions given below :

(i) The solution must satisfy all the supply and demand constraints. In other words, the solution must be feasible.

(ii) The number of allocation, which are feasible m cost be m+n–1.

One important point to be noted here is that, any solution which satisfies above two conditions is called a non-degenerate basci feasible solution & otherwise, the solution will be called degenerate solution.

Step 3 : In this step we test for the optimality of the solution. Here we shall discuss Modified Distribution Method (MOD_1) to test for the optimality of the solution obtained in step 2. If our solution is optimal, we shall step here. Otherwise, we have to determine a new improved solution repeating step 3.

2.2 Objectives :

After going through this unit students will be able to understand.

(i) how to establish an initial basic feasible solution by three different methods.

(ii) how to test an initial basic feasible solution for optimately by MODI method.

(iii) what is the difference between a transportation problem and an assignment problem.

2.3 Establishing an Initial Basic Feasible Solution :

An initial basic feasible solutions is one which satisfies all the supply and demand constraints (also called the nine conditions). There are several methods available to establish an initial basic feasible solution. Here we shall discuss three different methods.

1. North-West Corner Rule : It is a simple and efficient method to obtain an initial basic feasible solution. The method can be summarised in the following steps.

Step 1 : Start with the cell (1, 1) at the north west corner of the transportation matrix and allocate maximum possible limit, i.e. minimum of the rim values for the first row and first column, i.e. min (a_1, b_1).

Step 2 :

(a) If the allocation made in cell (1, 1) is equal to the supply available (i, e, a_1) there more down to cell (2, 1) and repeat the step 1. again

(b) If the allocation made in cell (1, 1) is equal to the demand of the first destination (i, e, b_1), there more to cell (1, 2) and repeat the step 1 again.

(c) If $a_1=b_1$, then we allocate x_1,$=a_1$ or b_1 and more diagonally to cell (2, 2).

Step 3 : Repeat the procedure until all the available quantity is exhausted, i.e. till an allocation is made in the south-east corner cell of the transportation table.

2.4 Illustrated Example on North West Corner Rule :

Ex. 1. Determine an initial basic feasible solution to the following transportation problem using North West Corner Rule.

	D_1	D_2	D_3	D_4	
O_1	6	4	1	3	14
O_2	8	9	2	7	16
O_3	4	3	6	1	5
	6	10	15	4	35

Availability

Requirements

where, O_i and D_j represents i^{th} origin and j^{th} destination respectively.

Sol^n : The given transportation table has 12 cells. Following North West Corner Rule, we make the first allocation in the cell (1, 1). Comparing the rim values of row 1 and column 1, we allocate min (14, 6)=6 to this cell. Following step 2(b) the second allocation is made at the cell (1, 2) and its magnitude is min (14–6, 10)=8.

Now the third allocation is made in the cell (2, 2) following step 2(a) and the alloted amount is min (16, 10–8)=2. Now, the fourth allocation is made at the cell (2, 3), which is min (16–2, 15)=14. The fifth allocation is made in the cell (3, 3), amounting to min (5, 15–14)=1. The sixth and the last allocation is

	D_1	D_2	D_3	D_4	
O_1	[6] 6	[8] 4	1	3	14
O_2	8	[2] 9	[14] 2	7	16
O_3	4	3	[1] 6	[4] 1	5
	6	10	15	4	

made in the cell (3, 4), which is min (5–1, 4)=4.

Now, since all the rim conditions are satisfied and hence an initial solution is obtained. The complete allocation and resulting feasible solution is displayed in the following transportation table :

	To					
From	3	4	6	8	9	20
	2	10	1	5	8	30
	7	11	20	40	3	15
	2	1	9	14	16	13
	40	6	8	18	6	

The total transportation cost of the initial solution is obtained by multiplying the quantity x_{ij} in the occupied cells with the corresponding unit cost c_{ij} and adding. So, the transportation cost following the above rule is

Total cost x = 6 6+8 4+2 9+14 2+1 6+4 1

= 124

Note :

1. In the North West Corner Rule, we move to the right or down. So no loop (closed circuit) can be formed. Have we get a non-degenerate basic feasible solution.
2. However, the basic feasible solution obtained by this method may be far from optimum, as the method donot take into account the cost of transportation.

Ex. 2 : Determine the initial basic feasible solution to the following transportation problem using North-West Corner Rule.

	To					
From	[20] 3	4	6	8	9	20
	[20] 2	[4] 10	[4] 1	5	8	30
	7	11	[4] 20	[11] 40	3	15
	2	1	9	[7] 14	[6] 16	13
	40	6	8	18	6	

Solution : Starting with the cell (1, 1), we allocate min (20, 4)=20 to this cell. That is 20 units of the commodity is to be shipped from origin 1 to destination 1. Though origin capacity is exhausted (rim condition fulfilled), destination 1 still require 20 units. Hence we allocate min (30, 40–20) to the cell (2, 1), since origin capacity is not exhausted, we move to cell (2, 2) and allocate min (30–26, 8)=4 to this cell.

Next we move to cell (2, 3) and allocate min (30–26, 8)=4 to this cell. Now the rim conditions for origin 2 and destination 2 is exhausted. Hence we move down to cell (3, 3) and allocate min (15, 8–4)=4 there and move horizontally to cell (3, 4). We allocate min (15–4, 18)=11 to this cell. Next we move down to cell (4, 4) and allocate min (13, 18–11)=7 to this cell. Our last allocation is in the cell (4, 5) and we allocate min (13–7, 6)=6 to this cell.

Now all the rim conditions are fulfilled and hence we arrive at a non-degenerate basic feasible solution, which is displayed in the following transportation table :

		To W_1	W_2	W_3	Availability
From	F_1	2	7	4	5
	F_2	3	3	1	8
	F_3	5	4	7	7
	F_4	1	6	2	14
Requirement		7	9	18	

The transportation cost following this route is given by,

$Z = 20\ \ 3+20\ \ 2+6\ \ 10+4\ \ 1+4\ \ 20+11\ \ 40+7\ \ 14+6\ \ 16$

$= 878$

2.5 Least Cost Method (Method of Matrix Hinema) :

This method takes into account minimum unit cost of transportation for obtaining initial solution. As the objective of a transportation problem is to minimize the transportation cost, we try to transport, as much as possible, through these cells, where unit cost of transportation is minimum. The method can be summarized as follows :

Step 1 : Select the cell with the lowest unit cost in the cost matrix of the transportation table and allocate as much as possible in this cell. Let the smallest cost be c_{ij}. We allocate $x_{ij}=\min(a_i, b_j)$ there.

Step 2 : If $x_{ij}=a_i$, cross off the 6^{th} row and decrease b_j by a_i and move to next step.

If $x_{ij}=b_j$, cross of the j^{th} column and decrease a_i by b_j and move to next step.

If $x_{ij}=a_i=b_j$, then cross off either the i^{th} row or j^{th} column and then move to step 3.

Step 3 : Repeat the procedure until all the rim conditions are satisfied, i.e. entire available supply and required demand is satisfied. If the minimum cost is not unique, select the cell where maximum allocature is possible.

Illustrated Examples :

1. Find the initial basic feasible solution of the following transportation problem by the Least Cost Method, whose cost matrix is given by,

	W_1	W_2	W_3	
F_1	2	7	4	5
			[8]	
F_2	3	3	1	8
F_3	5	4	7	7
F_4	1	6	2	14
	7	9	10	

Sol$_n$: The transportation problem given above has 12 cells. Following the matrix minima method, we examine the cost matrix carefully and find that lowest cost is in the cells (2, 3) and (4, 1). We choose the cell (2, 3) as we can allocate 8=min (8, 18), compared to the cell (4, 1), where we can allocate only 7=min (7, 14), we strike off the 2nd row and that leaves column W_3 with 18–8=10 units. Accordingly our cost matrix will be.

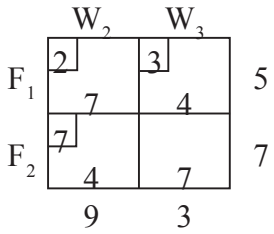
	W_1	W_2	W_3	
F_1	2	7	4	5
F_3	5	4	7	7
F_4	[7]			
	1	6	2	7
	7	9	10	

Next lowest cost 1 is in the cell (4, 1), where we can allocate min (7, 14)=7 units and strike off column W_1. That leaves 7 units with F_4.

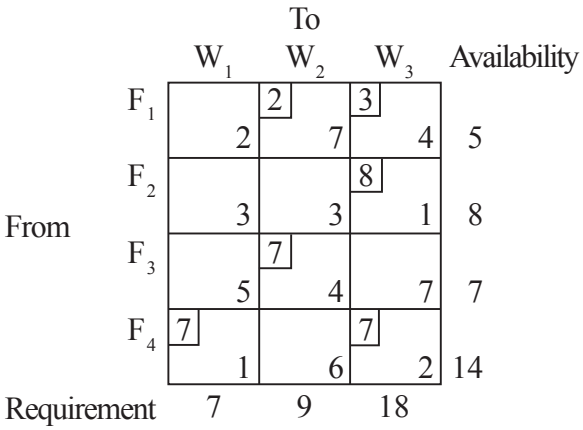
	W_2	W_3	
F_1	7	4	5
F_3	4	7	7
F_4		[7]	
	6	2	7
	9	3	

Next lowest cost lines in the cell (4, 3) and we allocate min (10,7)=7 to this cell and strike off row F_4. That leaves column W_3 with 3 units.

Next lowest cost is 3 in the cell (1, 3) where are allocate min (3, 5)=3 units, leaving F_1 with 2 units. Which we allocate to the cell (1, 2). Next to fulfill all the rim conditions we allocate 7 units to cell (2, 2).



Now, since all the dimand and supply is exhausted, the initial basic feasible solution isarrived at as



the total transportation cost is,

Rs (2 7+3 4+8 1+7 4+7 1+7 2)

=Rs 83

2. From the following cost matrix find the initial basic feasible solution using Least Cost Method and also calculate the total transportation cost.

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Sol^n : In the above cost matrix the lowest cost 8 is in the cell (3, 2), where we can allocate min (8, 18)=8 units and that meets the complete demand of D_2. But 18–8=10 units left with supplier S_3. Striking off the column D_2, the next lowest cost 10 lus in the cell (1, 4), where we can allocate min (7, 14)=7 units. Hence the capacity of S_1 is exhausted and leaves D_4 with 14–7=7 units.

Next lowest cost 20 is in the cell (3, 4), where we can allocate min (7, 10)=7 units and strike off the column D_3. That leaves S_3 with 3 units of supply.

Our next smallest cost is not unique, i.e. they lie in the cells (3, 1) and (2, 3), where the transportation cost is 40. We choose to allocate in the cell (2, 3) as we can allocate 7= min (7, 9), compared to 3= min (5, 3) in this cell. After striking off D_3, the supplier S_2 is left will only 3 units, which we allocate in the cell (3, 1).

The remaining 2 units is allocated to cell (2, 1) to meet the demand and supply conditions of D_1 and S_2 respectively. Now, since the demand and supply at each destination and origin is exhausted, we arrive at an initial basic feasible solution as shown below :

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	[7] 10	7
S_2	[2] 70	30	[7] 40	60	9
S_3	[3] 40	[8] 8	70	[7] 20	18
Demand	5	8	7	14	34

Hence the total transportation cost by LCM is given by,
Rs (7 10+2 70+7 40+3 40+8 8+7 20)
= Rs 814 ×

2.6 Vogel's Approximation Method (Unit Cost Penalty Method)

Vogel's approxination method of finding initial basic feasible solution is preferred to the two methods discussed so far. The North West corner Method doesnot yield optimal solution as it doesnot take into account the cell costs. The Least cost method provides a better solution by considering the cost of transportation and allocating as much as possible to the lowest cost cells. However, in Vogel's approximation Method, allocation are made on the basis of penalty cost that would have been incurred, if the allocation in certain cell with minimum transportation cost would have missed.

Basic steps in Vogel's Approximation Method are as follows :

Step 1 : Determine the penalties for each row as well as each solumn by calculating the difference between the smallest and the next-to-smallest cost in each row as well as in each column. Display these figures along with the transportation table.

Step 2 : Identify the row or column with largest penalty and allocate as much as possible, i.e. min (a_i, b_j) in the $(i, j)^{th}$ cell and cross off the i^{th} row or j^{th} column. If there is a tie in the values of the variables, select the cell where maximum allocation is possible.

Step 3 : Recompute the penalties for row and column for the reduced matrix as in step 1 and go to step 2. Repeat the procedure until the entire available demand at the sources and the total supply in the destination are satisfied.

Illustrated Example :

1. Obtain initial basic feasible solution of Vogel's Approximation method for the following cost matrix of a transportation table.

		To W_1	W_2	W_3	Availability
From	F_1	2	7	4	5
	F_2	3	3	1	8
	F_3	5	4	7	7
	F_4	1	6	2	14
Requirement		7	9	18	

Sol^n : First we calculate the differences between the smallest and next-to-smallest cost in each row as well as in each column and duplay then penalties along side the cost matrix as shown below :

	W_1	W_2	W_3	Availability	Penalty
F_1	2	7	4	5	(2)
F_2	3	3	[8] 1	8	(2) ←
F_3	5	4	7	7	(1)
F_4	1	6	2	14	(1)
Requirement	7	9	18		
Penalty	(1)	(1)	(1)		

↑

Here, the maximum penalty (2) is associated with F_1 and F_2. We can allocate 5=min (5, 7) in the lowest cost cell (1, 1) and 8=min (8, 18) to the cell (2, 3). Hence we allocate 8 to (2, 3) and strike off the row F_2. Our reduced matrix thus is,

	W_1	W_2	W_3	Availability	Penalty
F_1	2	7	4	5	(2)
F_3	5	4	7	7	(1)
F_4	1	6	[10] 2	14	(1)
Requirement	7	9	10		
Penalty	(1)	(2)	(2)		

↑

	W_1	W_2	Availability	Penalty
F_1	[5] 2	7	5	(5)
F_2	5	4	7	(1)
F_3	1	6	4	(5)
Requirement	7	9		
Penalty	(1)	(2)		

Now the maximum penalty 2 is associated with row F_1 and columns W_2 and W_3. We allocate maximum 10 units to the lowest cost 2 in W_3 and strike off that column. Hence, the reduced matrix takes the form,

	W_1	W_2	Availability	Penalty
F_3	5	4	7	(1)
F_4	[2] 1	6	4	(5)
Requirement	2	9		
Penalty	(4)	(2)		

Again, the maximum penalty 5 is associated with the rows F_1 and F_3 and we allocate maximum possible value 5 to the lowest cost 2 in F_1 and strike off F_1. Hence, the reduced matrix is,

	W_1	W_2	W_3	Availability
F_1	[5] 2	7	4	5
F_2		3 3	[8] 1	8
F_3	5	[7] 4	7	7
F_4	[2] 1	[2] 6	[10] 2	14
Requirement	7	9	18	

Since the maximum penalty (5) is associated with row F_4, we allocate 2 to the cell (4, 1) and strike off row F_4. Next we allocate 2 to the cell (F_4, W_2) and 7 to the cell (F_3, W_2), so that all demand and supply conditions are fulfilled. Eventially, the initial basic feasible solution is,

i.e. total transportation cost is,
Rs (5 2+8 1+7 4+2 1+2 6+10 2)
= Rs 80

Note : The transportation cost by the Vogel's approximation method is less than that associated with the initial basic feasible solution obtained by North West Corner Method or Least Cost Method.

2. Obtain the initial basic feasible solution by Vogel's Approximation Method for the following transportation problem with cost matrix :

	D_1	D_2	D_3	D_4	Availability
O_1	9	7	11	10	35
O_2	10	13	14	8	50
O_3	15	10	17	6	40
Requirement	45	20	30	30	

Sol^n : At first we calculate the penalties for each row as well as each column as before and place there alongside the transportation table. Then we allocate min (30, 4) in cell (3, 4) as against the minimum cost cell for maximum penalty (4) and consequently we get the following table

	D_1	D_2	D_3	D_4	Availability	Penalty
O_1	9	7	11	10	35	(2)
O_2	10	13	14	8	50	(2)
O_3	15	10	17	30 6	40	(4)
Requirement	45	20	30	30		
Penalty	(1)	(3)	(3)	(2)		

Next we strike off the column D_4 and again find the penalties for the reduced matrix.

	D_1	D_2	D_3	Availability	Penalty
O_1	9	7	11	35	(2)
O_2	10	13	14	50	(3)
O_3	15	10 10	17	10	(5)
Requirement	45	20	30		
Penalty	(1)	(3)	(3)		

In the above cost matrix we allocate min (20, 10)=10 to the cell (3, 2), against the minimum cost cell of Highest-penalty (5) and strike off O_3, our reduced cost matrix becomes,

	D_1	D_3	Availability	Penalty
O_3	9	25 11	25	(2)
O_3	45 10	5 14	50	(4)
Requirement	45	30		
Penalty	(1)	(3)		

Next we allocate min (45, 50) to the cell (2, 1), i.e. we allocate 45 to this cell. Next we allocate 5 to cell (2, 3) and 25 to the cell (1, 3), so that all the demand and supply conditions are fulfilled.

Eventually, the initial basic feasible solution is,

	D_1	D_2	D_3	D_4	Availability
O_1	9	10 7	25 11	10	35
O_2	45 10	13	5 14	8	≤ 50
O_3	150	10 10	17	30 6	40
Requirement	45	20	30	30	

Total cost of transportation is,

Rs (10 7+25 11+45 10+5 14+10 10+30 6)

=Rs 1090//

Check your progress

1. Obtain the initial basic feasible solution of the transportation problem whose cost matrix is given by,

		Destination A	B	C	D	Availability
Origin	I	1	5	3	3	34
	II	3	3	1	2	15
	III	0	2	2	3	12
	IV	2	7	2	4	19
Requirement		21	25	17	17	

Use NWCM, LCM and VAM to find the transportation cost in each ease and compare your results.

2.7 Test for Optimality

Once an initial basic feasible solution is obtained, the next step is to test for the optimality of the solution. An optimal solution is one, when there is no other alternate solution i.e. transportation routes, that will futher reduce the total transportation cost. We test for the optimality of a solution as follows :

Step 1 : For a basic feasible solution, first we determine a set of (m+n) numbers U_i (i=1, 2, ..., m) and V_j (j=1, 2, ..., n) called the dual variables such that for each occupied cell
(i, j), $c_{ij}=u_i+v_j$

Here we assign an arbitrary value to one of u_i's and v_j's, such that the rest (m+n–1) values can be easily calculated from the relation,

$c_{ij}=u_i+v_j$

Step 2 : Next we calculate net evaluation d_{ij} for each of the unoccupied cells (i, j) following the relation,

$d_{ij}= c_{ij}–(u_i+v_j)$

Step 3 :

Now (i) if all $d_{ij}>0$, the B.F.S. is optimal and unique.

(ii) if all d_{ij} 0, the B.F.S. is optimal but not unique.

(iii) if atleast one $d_{ij}<0$, then the solution is not optimal and proceed to step 4.

Step 4 : In this care we form a new basic feasible solution and in the new basic feasible solution we allocate maximum to the cell, where d_{ij} is– we and minimum, by making an occupied cell empty.

Step 5 : Then again we go through the steps 1 to 3 to test for optimality of this new basic feasible solution.

An efficient technique called the Modified Distribution (MODI) method, which helps in comparing the relature advantage of allocation of all the unoccupied cells simultaneously is discussed below. This method is based on the concept of 'Duality'.

2.8 Transportation Algorithm for Minimization Problem (MODI Method)

Transportation algorithm for minimization problem can be summarized as follows :

Step 1 : At first we construct a transportation table showing the capabilities $a_1, a_2, ..., a_m$ of the sources and the requirements $b_1, b_2, ...b_n$. Then we enter the costs c_{ij} at the upper left corner of the cells. Next we find the initial B.F.S. of the problem by any one of the methods discussed earlier and enter these allocations at the centre of the cells.

Step 2 : Next we find a set of values u_i(i=1, 2, ..., m) and u_j (j=1, 2, ..., n) as discussed in step 1 in the earlier section for each occupied cell (r, s), such that,

$c_{rs}=u_r+v_s$

Step 3 : Next an calculate (u_i+v_j) for each unoccupied cell and put it in the upper right corner of the corresponding cell.

Step 4 : Then we find the net evaluations,

$$d_{ij}=c_{ij}-(u_i+v_j)$$

for each' of these unoccupied cells and put these at the lower right corner of the cells.

Step 5 : Now depending on the values of the net evaluations we conclude as follows :
(i) if all $d_{ij}>0$, the solution is optimal and unique.
(ii) if all d_{ij} 0 and atleast one $d_{ij}=0$, these the rel^n is optimal but not unique.
(iii) if atleast one $d_{ij}<0$, then the solution is not optimal and we proceed to next step.

Step 6 : Next we form a new basic feasible solution by allocating maximum to the cell for which d_{ij} is minimum and negative, by making an unoccupied cell empty.

Step 7 : Again we repeat the steps 2 to step 5 to test for the optimality of this new basic feasible solution. We continue the process of empering the B.F.S. until an optimal solution is obtained.

Illustrated Examples

1. Goods are transported from four factores F_1, F_2, F_3 and F_4 to three godowns G_1, G_2, G_3. Determine the optimal transportation plan for the following transportation table :

Godown / Factory	G_1	G_2	G_3	≥Availability
F_1	19	15	30	6
F_2	14	16	25	10
F_3	23	12	16	12
F_4	11	21	39	15
Requirement	11	13	19	

Sol^n : At first we have to find the initial basic feasible solution by any one of the three methods discussed earlier in this chapter. We shall use Vogel's Approximation Method for this as shown below :

	G_1	G_2	G_3	Availability	Penalty
F_1	19	15	30	6	(4)
F_2	14	16	25	10	(2)
F_3	23	12	16	12	(4)
F_4	[11] 11	21	39	15	(10)
Requirement	11	13	19		
Penalty	(3)	(3)	(9)		

	G_1	G_2	Availability	Penalty
F_1	15	30	6	(15)
F_2	16	25	10	(9)
F_3	12	16	12	(4)
F_4	[4] 21	39	4	(18)
Requirement	13	19		
Penalty	(3)	(9)		

	G_1	G_2	Availability	Penalty
F_1	[4] 15	30	6	(15)
F_2	16	25	10	(9)
F_3	12	16	12	(4)
Requirement	9	19		
Penalty	(3)	(9)		

	G_2	G_3	Availability	Penalty
F_1	[3] 16	[7] 25	10	(9)
F_2	12	[12] 16	12	(4)
Requirement	3	19		
Penalty	(4)	(9)		

	G_1	G_2	G_3	Availability
F_1	(19)	(15) 6	(30)	6
F_2	(14)	(16) 3	(25) 7	10
F_3	(23)	(12)	(16) 12	12
F_4	(11) 11	(21) 13	(30) 19	15
Requirement	11	13	19	

Hence the initial B.F.S. is given by the following table :

the total transportation cost

Rs (6 15+3 16+7 25+12 6+11 11+4 21)

=Rs 710

Now to find the optimal solution we proceed as follows :

Step 2 : We determine a set of values u_i and v_j, such that for each occupied cell,

$C_{rs}=u_r+v_s$

$C_{12}=u_1+v_2=15$

$C_{22}=u_2+v_2=16$

$C_{23}=u_2+v_3=25$

$C_{33}=u_3+v_3=16$

$C_{41}=u_4+v_1=11$

$C_{42}=u_4+v_2=21$

Putting $u_1=0$ orbitrarly we get,

$u_2=1$ $v_1=5$

$u_3=-8$ $v_2=15$

$u_4=6$ $v_3=24$

Step 3 : Next we find cell evaluation u_i+v_j for each unoccupied cells (i, j) and enter at the upper right corner of the cell.

				Availability
	(19) (5)	(15) 6	(30) (24)	6
	(14) (6)	(16) 3	(25) 7	10
	(23) (3)	(12) (7)	(16) 12	12
	(11) 11	(21) 4	(39) (30)	15
Requirement	11	13	19	

Step 4 : Next we find the net evaluations,

$d_{ij}=c_{ij}-(u_i+v_j)$

and place it at the lower right corner of the corresponding cell

				U_i
(19) (5) (14)	(15) 6	(30) (24) (6)		0
(14 (6) (8)	(16) 3	(25) 7		1
(23) (3) (20)	(12) (7) (5)	(16) 12		–8
(11) 11	(21) 4	(39) (30) (9)		6
v_j 5	3	24		

Step 5 : Since all d_{ij} 70 in the above table, the B.F.S. shown in the table is the optimal solution and the optimum transportation cost = Rs 710\-

Ex. 2 : A company has factors F_1, F_2, F_3 which supply to warehouses w_1, w_2, w_3. Weekly factory capacities are 200, 160 and 90 units whereas weekly warehouse requirements are 180, 120 and 15 units respectively. Shipping costs from factories to warehouses are shown in the following cost matrix.

		Warehouse W_1	W_2	W_3	∴ Availability
Factory	F_1	16	20	12	200
	F_2	14	8	18	100
	F_3	26	24	16	90
Requirement		180	120	150	

Determine an optimal solution to the above transportation problem so that the total transportation cost is minimized.

Sol^n : At first we obtain initial basic feasible solution to the above transportation table as given in the steps below :

	W_1	W_2	W_3	Availability	Penalty
F_1	16	20	12	200	(4)
F_2	14	120 8	18	160	(6)
F_3	20	24	16	90	(8)
Requirement	180	120	150		
Penalty	(2)	(12)	(4)		

	W_1	W_2	Availability	Penalty
F_1	16	12	200	(4)
F_2	14	18	40	(4)
F_3	26	90 16	90	(10)
Requirement	180	150		
Penalty	(2)	(4)		

	W_1	W_2	Availability	Penalty
F_1	140 16	60 12	200	(4)
F_2	40 14	18	40	(4)
Requirement	180	60		
Penalty	(2)	(6)		

Hence the initial basic feasible solution of the given transportation problem is given below as :

	W_1	W_2	W_3	Availability
F_1	140 16	20	60 12	200
F_2	40 14	120 8	18	160
F_3	26	24	90 16	90
Requirement	180	120	150	

and the total transportation cost associated with the sol^n is
Rs (16 140+12 60+14 40+8 120+16 90)
= Rs 5920

Step 2 : Next we determine the values of u_i and v_j, such that $_{ij}=u_i+v_j$, for each occupied cell

$c_{11}= u_1+v_1=16$ $c_{22}=u_2+v_2=8$
$c_{13}=u_1+v_3=12$ $c_{33}=u_3+v_3=16$

$c_{21}=u_2+v_1=8$

Assigning $u_1=0$, we get the values of u_i and v_j as

$u_1=0 \quad v_1=16$
$u_2=-2 \quad v_2=10$
$u_3=4 \quad v_3=12$

Step 3 : Next we obtain u_i+v_j for each unoccupied cell (i, j) and entire at the upper right corner of the cell and find the net evaluations,

$d_{ij}=c_{ij}-(u_i+v_j)$

for each of these cells and place these at the lower right corner of the cell as follows :

	W_1	W_2	W_3	Availability
F_1	(16) 140	(20) (10) (10)	(12) 60	200
	(14) 40	(8) 120	(18) (10) (8)	100
	(26) (20) (6)	(24) (14) (10)	(16) 9	90
Requirement	180	120	150	

Since all the net evaluations $d_{ij}>0$, the basic feasible solution given above is an optimal one.

the optimal solution is,

Transport 140 units from Factory F_1 to warehouse W_1
Transport 60 units from Factory F_1 to warehouse W_3
Transport 40 units from Factory F_2 to warehouse W_1
Transport 120 units from Factory F_2 to warehouse W_2
Transport 90 units from Factory F_3 to warehouse W_3

Ex. 3: Obtain an optimal solution for the following transportation problem by MODI method.

				Supply
	19	30	50	10	7
	70	30	40	60	9
	40	8	70	20	18
Demand	5	8	7	14	

Sol^n : At first we obtain an initial basic feasible solution to the above transportation problem by Vogel's approximation method and is displayed in the table below :

	D_1	D_2	D_3	D_4	Supply
S_1	(19) 5	(30)	(50)	(10) 2	7
S_2	(70)	(30)	(40) 7	(60) 2	9
S_3	(40)	(8) 8	(70)	(20) 10	18
Demand	5	8	7	14	

Here, total number of occupied cells = m+n–1=6 and hence initial basic feasible relation is non-degenerate. Thus an optimal solution can be obtimal.
The total transportation cost is,
Rs (19 5+10 2+40 7+60 2+8 8+20 10)
=Rs 779

Step 2 : Next we obtain the values of u_i's and v_j's such that for each occupied cell,

$c_{rs}=u_r+u_s$

i.e. $c_{11}=u_1+v_1=19$ $c_{24}=u_2+v_4=60$

$c_{14}=u_1+v_4=10$ $c_{32}=u_3+v_2=8$

$c_{23}=u_2+v_3=40$ $c_{34}=u_3+v_4=20$

Putting $u_4=0$ orbitrarily we get,

$u_1=10$ $v_1=9$

$u_2=60$ $v_2=–12$

$u_3=20$ $v_3=–20$

Step 3 : Now we determine (u_i+v_j) for each unoccupied cell and put it in the upper right corner and also calculate $d_{ij}=c_{ij}–(u_i+v_j)$ for each of these cells and place it at the lower right corner of these cells.

	D_1	D_2	D_3	D_4	Supply
S_1	(19) 5	(30) (-2) (32)	(50) (-10) (60)	(10) 2	7
S_2	(70) (69) (1)	(30) (40) (-10)	(40) 7	(6) 2	9
S_3	(40) (29) (11)	(8) 8	(70) (0) (70)	(20) 10	18
Demand	5	8	7	14	

Step 4 : Since, in the above table, all d_{ij} are not zero or positive, the above solution is not optimal.

Step 5 : Since maximum –ve d_{ij}=–18, w give maximum allocation to this cell from an occupied cell and make necessary changes in other allocations as shown in the following table :

	D_1	D_2	D_3	D_4
S_1	5			2
S_2		$+\theta$ ↓	7	2–
S_3		8–		10+

Step 6 : Now the minimum allocation containing $-\theta$ is Q– .

Taking, 2– =0

=2

Hence the value of the allocation will also change. That is =2 will be added to cells (S_2, D_2) and (S_3, D_2) and will be subtracted from the cells (S_2, D_4) and (S_3, D_2). The new basic feasible solution is as shown in the following table.

	D_1	D_2	D_3	D_4	Supply
S_1	(19) 5	(30)	(50)	(10) 2	7
S_2	(70)	(30) 2	(40) 7	(60)	9
S_3	(40)	(8) 6	(70)	(20) 12	18
Demand	5	8	7	14	

Accordingly, the transportation cost for this solution is,

Rs (5 19+2 10+2 30+7 40+6 8+12 20)

=Rs 743

Which is lessthan that for initial basic feasible solution.

Step 7 : Again we test for the optimality of this basic feasible solution as in step 2.

i.e. $c_{11}=u_1+v_1=19$ $c_{23}=u_2+v_3=40$

$c_{14}=u_1+v_4=10$ $c_{32}=u_3+v_2=8$

$c_{22}=u_2+v_2=30$ $c_{34}=u_3+v_4=20$

Putting $u_1=0$, we get,

$u_2=32$ $v_1=19$

$u_3=10$ $v_2=-2$

$v_3=8$

$v_4=10$

Step 8 : Following step 3, we get the following table :

	D_1	D_2	D_3	D_4	Supply
S_1	(19) 5	(30) (-2) (32)	(50) (8) (42)	(10) 2	7
S_2	(70) (51) (19)	(30) 2	(40) 7	(60) (42) (18)	9
S_3	(40) (29) (11)	(8) 6	(70) (18) (52)	(20) 12	18
Demand	5	8	7	14	

Here since all d_{ij} are+ue, the current solution is the optimal one with miinimum total transportation cost Rs 743//

Ex4 : Obtain the optimal solution by MODI method for the following transportation problem.

		To D_1	D_2	D_3	Supply
From	O_1	2	7	4	5
	O_2	3	3	1	8
	O_3	5	4	7	7
	O_4	1	6	2	14
Demand		7	9	18	

Sol^n : The initial basic feasible solution is obtained by Vogel's Approximation Method is as follows :

	D_1	D_2	D_3	Supply
O_1	(2) 5	(7)	(4)	5
O_2	(3)	(3)	(1) 8	8 ×
O_3	(5)	(4) 7	(7)	7
O_4	(1) 2	(6) 2	(2) 10	14
Demand	7	9	19	

Total transportation cost is
Rs (2 5+1 8+7 4+2 6+2 6+2 10)
=Rs 80

Here total number of occupied cells=m+n–1=3+4–1=6. Hence the initial basic feasible solution is non-degenerate and hence an optimal basic feasible solution exist.

Step 2 : We obtain u_i's and v_j's, such that for each occupied cell (r, s), $c_{rs}=u_r+v_s$

$c_{11}=u_1+v_1=2$ $c_{41}=u_4+v_1=1$
$c_{23}=u_2+v_3=1$ $c_{42}=u_4+v_2=6$

$c_{32}=u_3+v_2=4$ $c_{43}=u_4+v_3=2$

Putting $u_4=0$, orbitrarily we get,

$u_1=0$ $v_1=1$

$u_2=-1$ $v_2=6$

$u_3=-2$ $v_3=2$

$u_4=0$

Step 3 : Next we calculate (u_i+v_j) and $d_{ij}=c_{ij}-(u_i+v_j)$ for each unoccupied cell and place there in their respective positions, as shown in the following table :

	D_1	D_2	D_3	Supply
O_1	(2) 5	(7) (7) (0)	(4) (3) (1)	5
O_2	(3) (0) (3)	(3) (5) (-2)	(1) 7	8
O_3	(5) (-1) (6)	(4) 7	(7) (0) (7)	7
O_4	(1) 2	(6) 2	(2) 10	14
Demand	7	9	18	

Step 4 : Since $d_{22}=-2<0$, so the solution under consideration is not optimal one. We give allocation to the cell c_{22} from an unoccupied cell and make the necessary changes as shown in the following table:

	D_1	D_2	D_3
O_1	5		
O_2		+	8–
O_3			
O_4	2	2–	10+

Since minimum allocation containing – is 2– , we take 2– =0

=2

Step 5 : The new basic feasible solution is as shown in the following table :

	D_1	D_2	D_3	Supply
O_1	(2) 5	(7)	(4)	5
O_2	(5)	(3) 2	(1) 6	8
O_3	(5)	(4) 7	(7)	7
O_4	(1) 2	(6)	(2) 12	14
Demand	7	9	18	

and the total transportation cost for this is,
Rs (2 5+2 3+7 4+2 1+12 2)
=Rs 76
Which is less than that is step 1.

Step 6 : Again proceeding in usual manner we find the values of u_i and v_j as follows :

$c_{11}=u_1+v_1=2$ $c_{32}=u_3+v_2=4$
$c_{22}=u_2+v_2=3$ $c_{41}=u_4+v_1=1$
$c_{23}=u_2+v_3=1$ $c_{43}=u_4+v_3=2$

Taking $u_2=0$, we get,

$u_1=2$ $v_1=0$
$u_2=0$ $v_2=3$
$u_3=1$ $v_3=1$
$u_4=1$

Step 7 : Next we calculate (u_i+v_j) and $d_{ij}=c_{ij}-(u_i+v_j)$ for unoccupied cells and place there in their respective positions. Accordingly we get,

	D_1	D_2	D_3	Supply
O_1	(2) 5	(7) (5) (2)	(4) (3) (1)	5
O_2	(5) (0) (5)	(3) 2	(1) 6	8
O_3	(5) (1) (4)	(4) 7	(7) (2) (5)	7
O_4	(1) 2	(6) (4) (2)	(2) 12	14
Demand	7	9	18	

Since all $d_{ij}>0$, hence the current basic feasible solution is the optimal one. Thus solution to the transportation problem is,

Transport 5 units from source O_1 to destination D_1
Transport 2 units from source O_2 to destination D_2
Transport 7 units from source O_3 to destination D_2
Transport 2 units from source O_4 to destination D_1
Transport 12 units from source O_4 to destination D_3
and the total transportation cost is Rs 76.

Check your progress

1. A steel company has three open heart furnaces and five rolling mills-Transportation cost (Rs per quintal) for shipping steel from furnaces to rolling mills are shown in the following table :

		Mills					
		M_1	M_2	M_3	M_4	M_5	Capacities (in quintal)
Famous	F_1	4	2	3	2	6	8
	F_2	5	4	5	2	1	12
	F_3	6	5	4	7	3	14
Requirement (in quintal)		4	4	6	8	8	

What is an optimal shipping schedule?

2. Use VAM be find the initial basic feasible solution :

		Destination				
		I	II	III	IV	Availability
Origin	1	5	8	3	6	30
	2	4	5	7	4	50
	3	6	2	4	6	20
Requirement		30	40	20	10	

Test this solution for optimality.

Difference between a transportation problem an assignment problem :

The assignment problem may be seen as a special case of transportation problem, where each origin is associated with one and only one destination. In the light of the explanations so far, we may consider an assignment problem as a special case of transportation problem where m=n, all a_i's and b_j's are unity and each x_{ij} can take value 0 or 1 only. Under such circumstances, exactly n of the x_{ij}'s can be non zerom, i.e. 1 in this case, one for each origin and one for each destination.

2.9 References:

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Block 3 : Theory of Games and Queing Theory
Unit 1

Learning outcomes : After going through this chapter, students will be able to,

(i) distinguish a game situation.

(ii) understand the concepts of players, strategy, pay off and pay off matrix.

(iii) distinguish between pure strategy and mixed strategy

(iv) learn what is zero-sum game.

Introduction : In economics, the term 'game' refers to the general situation of conflict, where two or more opposite parties are in valued in decision making activities, where the action of one depends upon the action taken by the opponent. The parties of conflicting interests are termed as players. The player in the theory of games may be single individual, a group of individuals or an organisation. Thus game theory provides a theoritical frame work for right choice of situation among competition players.

A great variety of competition is generally seen in our everyday life. For example, candidates fighting in an election may be referred to as players, because there is conflict of interest among them. Here, the aim of each candidate is to assure as many votes as possible, beating all other candidates. similarly competitive situations arise in situations like :

(i) pricing of products, where the price of a particular product is not only determined by the pricing policy of the firm alone, but also on the price level of the same product offered by different firms.

(ii) the success of a programme of a particular T.V. channel depends mainly on what the competitive channels show at that particular time shot.

(iii) The success of an advertising agency is also dependent in what the other advertising agencies will offer to its customers.

Thus, what we see is that, game theory is a type of decission making strategy where one's course of action doesnot depend on the decision of one party alone, but is determined after taking into account all possible alternatives available to an opponent, playing the same game. Thus game theoty provides an opportunity to a player not only to evaluate his own course of action, but also to evaluate the courses of action of his opponents, so that he can win the game. It is assumed that, each opponent in a particular game is going to play in a rational manner, resolving the conflict in his favour to win the game.

Origin and Development of Game Theory :

The mathematical analysis of Game theory is based upon the first important result of game theory, called the minimax theorem, provided the Mathematician J. Von. Neumann, in the year 1928, also known as the father of Game theory. According to this theorem, each player will try to maximize his minimam gain or minimize his maximum loss. In his book titled "Theory and Practices of Games and Economic Behaviour", which he co-authored with Australlian economist Moxgenstern is considered as a pioneer work by experts all over the world, since that time the games theory has developed as a result of co-operation between mathematicians and economists.

However, the game theory is capable of analysing simple competitive situations only, it cannot handle what is really needed in more complex situations in industry or elsewhere. So there is always a gap between the theortical concept and formed application in solving real life situations.

Terminology of Game theory :

(i) Players : The participants of a game, who act as decision makers are termed as players. A game requires atleast two players. A game having only two players is called a two person game. If the number

of players is more than two, it is called 'n-person game'.

(ii) Strategy : The strategy for a player is the list of all possible courses of action available, that he may take for every outcome. It is assumed that the rules governing the choices as well as outcome that may result from a particular choice are known and expressed in numerical values in advance by the players. But the players need not necessarily have definite information regarding each other's strategy.

To ellustrate it little further, let us assume that MrX, a manufacturer is facing a problem of deciding whether he should reduce the price of his product or should maintain the same price, so that he can counteract competition and make maximum profit. In this care he has two strategies, viz,

(i) reduce price

(ii) maintain the price

Similarly his competitors have also two strategies, any one one of which he may employ to counteract Mr. X. They are,

(i) maintain price

(ii) reduce price

A particular strategy by which a player can optimize his gain or losses, without having any knowledge about his competitors strategies is called the optimal strategy. That is, it is the strategy that puts the player in most adventageous position. The expected outcome per play, where the players play their optimal trategy is called the value of the game. Any deviation from the optimal strategy will reduce the payoff. The strategies can be classified as pure strategy and mixed strategy.

(i) Pure Strategy : It is the deterministic situation, i.e. here a player known exactly what the other player is going to do so, in this case the decision rule is to select a particular course of action or the same strategy, regardless of the other player's strategy and the objective of the player is to maximize his profit or minimze his losses.

(ii) Mixed Strategy : Mixed strategy is a decission, which the player chooses in advance of each play, with a particular probability law. The mixed strategy is more advantageous then pure strategy as here the opponent keep on guessing as to which course of action is to be selected by the opponents at each play. It should be noted here that pure strategy is a particular case of mixed strategy.

(iii) Play : A play is said to be played when each player chooses one of his strategies. The assumptions made in a play are :

(a) the choice of strategy by the players is made simultaneously.

(b) No player knows his opponents choice until he has decided his own course of action.

(iv) Pay off : It is the outcome of the game, in terms of gains or losses. Every combination of strategy of players determines an outcome, called the payoff. For example, we may consider the care of Mr. X and his opponent as explained earlier, where both the players have two strategies namely, 'maintain price' and 'reduce price'. Here all possible combinations of strategies will be,

a. X reduces his price while his opponent keeps price constant.

b. X reduces his price and opponent also reduces price.

c. X keeps price constant, opponent reduces his price.

d. X keeps his price constant and opponent also keeps price constant.

Each of the above combinations, which represents gain (loss) to a player is called the payoff.

(v) Pay off matrix : The gains or losses resulting from a game can be represented in the form of a matrix, called the payoff matrix.

It the player A has m strategies $A_1, A_2, ..., A_m$ and the player B has n strategies $B_1, B_2, ..., B_n$, then the total number of possible outcomes is $m \times n$. Here m need not be equal to n. It is assumed that a player knows not only his all courses of action, but also of his opponents. Let a_{ij} is the payoff to A, in

A's payoff matrix, when the player A chooses the course of action i and B chooses the course of action j. A's payoff matrix is shown below :

Player B's strategy

Player A's strategy	B_1	B_2	----	B_j	----	B_2
A_1	a_{11}	a_{12}	----	a_{ij}	----	a_{1n}
A_2	a_{21}	a_{22}	----	a_{2j}	----	a_{2n}
.	----------					
.	----------					
A_i	a_{i1}	a_{i2}	----	a_{ij}	----	a_{in}
.	----------					
A_m	a_{m1}	a_{m2}	----	a_{mj}	----	a_{mn}

Where it is assumed that the player A is always the gainer. So, he will wish to gain as large a_{ij} as possible, while the action of the player B will be to make this a_{ij} as small as possible. Obviously, gain to player B is $-a_{ij}$, which is loss to player A. That is B's payoff matrix is,

Player B's strategy

Player A's strategy	B_1	B_2	----	B_j	----	B_2
A_1	$-a_{11}$	$-a_{12}$	----	$-a_{ij}$	----	$-a_{1n}$
A_2	$-a_{21}$	$-a_{22}$	----	$-a_{2j}$	----	$-a_{2n}$
.	----------					
.	----------					
A_i	$-a_{i1}$	$-a_{i2}$	----	$-a_{ij}$	----	$-a_{in}$
.	----------					
A_m	$-a_{m1}$	$-a_{m2}$	----	$-a_{mj}$	----	$-a_{mn}$

(vi) Finite and Infinite games : A game is said to be a finite game if it has a finite number of moves, each of which has a finite number of allimatives. A game which is not a finite game is called an infinite game.

(vii) Zero-sum game : A game is said to be a zero sum game, if the sum of payments of all players or competitors after a play is equal to zero.

Suppose, two players A and B play a game called 'two finger move'. Both the players A and B show either one or two fingers. If the number of fingers of player A coincides with the number of fingers shown by the player B, then the player A wins and gets Rs 10 from player B. Again, if the number of fingers of both the players doesnot match, then the player B wins and he gets Rs 10 from player A. For this game the payoff matrix for the player A will be as follows.

		Player B	
		finger I	finger II
Player A	finger I	10	–10
	finger II	–10	10

The game shown above is a two person zero sum game, since the winning of one player is equal to the loss for the other. In B's payoff matrix, cell entries will be just the negative of the corresponding cell entries of A's payoff matrix.

Check your progress

1. What is a game in the game theory. What are the properties of a game?
2. What are the assumptions mode in the theory of games?
3. Differentiate between pure strategy and mixed strategy in the theory of games.
4. Define : (i) Payoff matrix, (ii) zero-sum game, (iii) strategy

Block 3 : Theory of Geneus and Queeing Theory
Unit 2 : Two person Zero–Sum Game

Learning objective : After going through this unit students will be able to

(i) understand when a zero-sum game is called a two person zero-sum game.

(ii) find out the optimal strategy for both the players using maximum minimax principle of optimality.

Two Person Zero sum Game

Introduction :

As discussed in the previous chapter, games having zero-sum character, i.e. the algebric sum of gains and loves of all the players is zero is called a zero-sum game. The game doesnot add a single penny to the initial money, with which the game gets stated. There is only a new destribution of that money among the players, when only two players are involved in such a game, it is called a two person zero-sum game. In other words, a game is said to be a two person zero-sum game, if one player's gain is equal to the other player's loss, so that the total sum is equal to zero. The basic assumptions of a two person zero-sum game are :

(i) there should be exactly two players with opposite interests.

(ii) this should be finite number of strategies available to each player, which may or may not be same for both the players.

(iii) for any strategy selected by a player, there should be a payoff.

(iv) one player's gain is exactly equal to the other player's loss.

Payoff matrex of a Two Person zero-sum game

We have already discussed about the payoff of a game. It is nothing but the quantitative measure of satisfaction of a player at the end of a play. These values of gains or losses when represented in a matrix form is termed as payoff matrix. In a two person zero-sum game, as the gain of one player is just the loss of the other player or vice versa, the payoff matrix of one player will be the same as the other player's payoff, with only sign changed.

Let us consider a two person coin tossing game. Both the players A and B toss the coin rein ultaneously. Player B will pay Rs 10 to player A if 'Head' occurs in both the coins and Rs 20 if both the coins show 'Tail'. Otherwise, the player A will pay Rs 5 to player B. Here, each player A and B can choose between any of the two pure strategies H (Head) and T (Tail). We can call this a two person zero-sum game, because winning of A means loss for B or vice versa. If we want to express the about informations in terms of payoff for the player A it will take the following form.

		Player B	
		H	T
Player A	H	10	–5
	T	–5	20

similarly B's payoff matrix will be just the series with all entries having a–w sign, i.e.

		Player A	
		H	T
Player B	H	–10	5
	T	5	–20

Maximum and Minimax Criterion of Optimality

In a game theory the but strategy for each player is determined by the maximin and minimax criterion of optimality. This criterion states that if a player lists the worst possible outcomes of all his potential strategies, he should choose that strategy which corresponds to the best of the worst situations. Such a strategy is called the optimal strategy.

Suppose, player A's payoff matrix is given by,

		Player B	
		1	2--------j---------n
Player A	1	a_{11}	a_{12}-----a_{ij}---------a_{1n}
	2	a_{21}	a_{22}-----a_{2j}--------a_{2n}
	-	----------	----------
	-	----------	----------
	i	a_{i1}	a_{i2}------a_{ij}--------a_{in}
	-	----------	----------
	m	a_{m1}	a_{m2}------a_{mj}-------a_{mn}

If A plays strategy 1 then his guranteed gain is at least $\underset{j}{\text{Min}}\ a_{1j}$

If a plays strategy 2 then his guranteed gain is at least $\underset{j}{\text{Min}}\ a_{2j}$

If A plays strategy i then his guranteed gain is at least $\underset{j}{\text{Min}}\ a_{ij}$

If A plays strategy m then his guranteed gain is at least $\underset{j}{\text{Min}}\ a_{mj}$

Now, according to Maximin, Minimax criterion, the player

A will choose that strategy which corresponds to the but of these worst situations, i.e. but of the outcomes

$\underset{j}{\text{Min}}\ a_{ij}, \underset{j}{\text{Min}}\ a_{2j}, \text{----------}, \underset{j}{\text{Min}}\ a_{mj}$

Thus, Maximin of A is given by $\underset{j}{\text{Min}}\left[\underset{j}{\text{Min}}\ (a_{ij})\right]$.

Similarly player B will try to choose the strategy that corresponds to the best of the worst situations.

If the player B plays strategy 1 he can not loose more than $\underset{i}{\text{Max}}\,(a_{i1})$

If the player B plays strategy 2 he cannot loose more then $\underset{i}{\text{Max}}\,(a_{i2})$

If the player B plays strategy j he cannot loose more than $\underset{i}{\text{Max}}\,(a_{ij})$

If the player B plays strategy n he cannot loose more than $\underset{i}{\text{Max}}\,(a_{in})$

According to Maximin Minimax criterion of optimality,

B will obviously choose that strategy which can gurantee him minimum of these maximin losses. That is

$\underset{i}{\text{Max}}\,(a_{i1})$, $\underset{i}{\text{Max}}\,(a_{i2})$, ------ $\underset{i}{\text{Max}}\,(a_{in})$

That is Maximax value for B is given by,

$\underset{j}{\text{Max}}\left[\underset{i}{\text{Max}}\ (a_{ij})\right]$ $\underset{j}{\text{Max}}$

Let, $\underset{j}{\text{Max}}\left[\underset{i}{\text{Min}}\ (a_{ij})\right]=a_{pq}$ — (1)

$\underset{i}{\text{Min}}\,[\text{Max}\,(a_{ij})]=a_{rs}$ — (2)

From (1) we see that a_{pq} is the smallest than any other element in the p^{th} row.

i.e. $a_{pq} \le a_{ps}$ — (3)

Where a_{ps} is any other element in the p^{th} row. Again equation (2) suggests that a_{rs} is the maximum element of the s^{th} column.

i.e. a_{rs} a_{ps} —(3)

(3) and (4) implus that,

a_{pq} a_{rs}

$\left[\underset{j}{\text{Max}}\ (a_{ij})\right] \le \left[\underset{i}{\text{Max}}\ (a_{ij})\right]$

or Maximin for A ≤ Minimax for B.

Generally, the maximin for A is called the lower value of the game and is denoted by and the minimax for B is called the upper value of the game and is denoted by $\overline{V}$. If we represent the value of a game by

V, then it must satisfy the inequality,

$$\underline{V} \leq V$$

In case $\underline{V} = V = \overline{V}$, the game is said to have a solution, and the game is said to be a game with saddle point.

ẋ

Ex. Given a two person zero-sum game between two players A and B. The payoff matrix for the player A is given below. Find the optimal strategy.

		Player B		
		I	II	III
Player A	I	–1	0	8
	II	4	2	4
	III	7	0	–2

Sol^{n} : Here in a competitive game between two players A and B, the player A tries to obtain largest possible a_{ij} using one of his strategies (I, II, III). On the otherhand, player B is determined to make A's gain as minimum as possible, by the use of his three strategies (I, II, III). In this case, A is called the maximixing player and B is called the minimixing player.

		Player B			$\overline{V}$
		I	II	III	Row maxima
Player A	I	(–1)	0	[8]	–1
	II	4	[(2)]	4	2 → Maximin value ( )
	III	[7]	0	(–2)	–2
Column maxima		7	2	8	
			↓		

Minimax value ($\overline{V}$)

Here if the player A chooses his first strategy, there the player B may choose one of the three strategies (I, II, III). In this case the player A's guranteed gain can be at least –1, i.e.

min [–1, 0, 8]=–1

Similarly, when the player A plays his IInd and IIIrd strategy, the player B plays his strategy in such a way that A's gain is minimum.

i.e. min [4, 2, 4] = 2, when A plays strategy II

min [7, 0, –2]= –2, when A plays strategy III.

The minimum value in each row is shown in the above table by a circle around them. Now, A's action will be to maximize his minimum gain. i.e.

Max [–1, 2, –2] = 2

so the player A will opt for II[nd] strategy.

Player B, on the otherhand see that if he plays his pure strategy I, he may loose at the most 7.

i.e. max [–1, 4, 7]

Similarly, applying strategies II and III, the playe B may loose at the most 2 and 8 respectively. The action of the player B will be to choose the best strategy which will minimize his loss. i.e.

min [7, 2, 8]=2, given by strategy II.

The player A's strategy is called the maximin strategy and the corresponding value is called the maximin value or lower bound ($\underline{V}$) of the game.

$\therefore \quad = 2$

Player B's strategy will be called the minimax strategy and the corresponding value the minimax value or upper bound of the game ($\overline{V}$).

i.e. $\overline{V} = 2$

In this case, $\underset{i}{\text{Max}} \left[\underset{j}{\text{Max}} \quad (a_{ij}) \right] = \underset{j}{\text{Min}} \left[\underset{i}{\text{Max}} \quad (a_{ij}) \right]$

or $\underline{V} = \overline{V}$

Hence the corresponding strategies of the game, i.e. strategy II for player A and B are called the optimal strategy.

Ex. 2. Check whether the following games have raddle point or not.

(i)

		B			$\underline{V}$
		I	II	III	
A	I	6	8	4	
	II	4	12	2	

(ii)

		B		
		I	II	III
A	I	3	–4	8
	II	–8	5	–6
	III	6	–7	6

Sol^{n} :

		B I	B II	B III	Row minima
A	I	[6]	8	(4)	4 →
	II	4	[12]	(2)	2
Column maxima		6	12	4 ↓ $\overline{V}$	

As discussed in example 1,

$$\max_i = 4, \quad \min_j \left[\max_i (a_{ij}) \right] = 4$$

$$\therefore \quad = \min_j \left[\max_i (a_{ij}) \right]$$

or $\underline{V} = \overline{V}$

$\therefore$ the game has a saddle point and the optimal strategy for the player A is I and for the player B, it is III.

(ii)

		B I	B II	B III	Row minima $\min_j \left[\max_i (a_{ij}) \right]$
A	I	3	(–4)	[8]	–4 →
	II	(–8)	[5]	–6	–8
		[6]	(–7)	6	–7
Column maxima		6	5 ↓ $\overline{V}$	8	

Here, $= -4$

$$\min_j \left[\max_i (a_{ij}) \right] = 5$$

since $\underline{V} \neq$, the game has no saddle point.

Check your progress

1. What is two person zero-sum game?
2. Explain two person zero-sum game with suitable example.
3. Explain 'best strategy' on the basis of minimax criterion of optimality.
4. Describe the maximin, minimax principle of game theory. What do you understand by saddle point.
5. Apply Maximin minimax principle to obtain the optimum strategies for the players in the following games.

(i)

$$\begin{array}{c} \\ A \end{array} \begin{array}{c} \begin{array}{ccc} B & & \end{array} \\ \begin{bmatrix} -2 & 15 & -2 \\ -5 & -6 & -4 \\ -5 & 20 & -8 \end{bmatrix} \end{array}$$

(ii)

$$\begin{array}{c} \\ \lambda \end{array} \begin{array}{c} \begin{array}{ccc} & B & \\ I & II & III \end{array} \\ \begin{array}{c} I \\ II \\ III \end{array} \begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix} \end{array}$$

Block 3 :
Unit III

After going through the chapter students will be able to

(i) solve a game with saddle point.
(ii) understand what is a strictly determinable game.
(iii) find optimal strategy for a game without a saddle point.
(iv) understand the concept of dominance in the solution of rectangular games.

Saddle Point : As discussed in the previous chapter, the saddle point of a payoff matrix is that point where maximum of the row minima and the minimum of the column maxima coincides. The payoff at the saddle point is called the value of the game. Mathematically, if a payoff matrix $\{a_{ij}\}$ is such that

$$\max_i \left[\min_j \{a_{ij}\} \right] = \min_j \left[\max_i \{a_{ij}\} \right] = Q_{rs} \text{ (ray)}$$

then the matrix is said to have a saddle point. Thus in a game with saddle point, the players use the same strategy throughout the game, or we can say that both the players use pure strategy.

A point to be noted here is that, the saddle point and hence the value of the game need not be unique. In general, the value of the game is denoted by v. If the payoff matrix $\{a_{ij}\}$ has the saddle point (r, s), then the corresponding strategies are called the optimal strategies. That is, in this case, the r^{th} strategy is the optimal strategy for one player and s^{th} strategy is the optimal strategy for the other player.

Thus to solve a game we need to look for the saddle point of the game. If the saddle point exists, the game is solved. But, unfortunately most of the payoff matrices is not possess any saddle point. In general the value of a game, v, satisfies the following inequality.

maximin value $\leq \vee \leq$ minimax value

or denoting the maximin value by and minimax value by $\overline{\vee}$ we get,

A game is said to be strictly determinable if,

$\overline{\vee} = \underline{\vee} = \vee$

and the game is said to be fair of

$\overline{\vee} = \underline{\vee} = 0$

Note : A saddle point of a payoof matrix is sometimes called the equilibrium point of the payoff matrix.

Theorem : Let $\{a_{ij}\}$ be the payoff matrix of a two person zero sum game. If $\underline{\vee}$ and $\overline{\vee}$ denote the maximin and minimax value of a game, then,

i.e. $$\min_j \left[\max_i \{a_{ij}\} \right] \geq \max_i \left[\min_j \{a_{ij}\} \right]$$

Proof : We know that, $\max_i \{a_{ij}\} \geq a_{ij} \forall j$

and $\min_j \{a_{ij}\} \leq a_{ij} \forall i$

suppose that,

and $\min_j \{a_{ij}\} = a_{ij^*}$

$\therefore$

If follows that, $\min_j \{a_{*ij}\} \geq a_{ij} \geq \max_i \{a_{ij^*}\}$

$\Rightarrow$

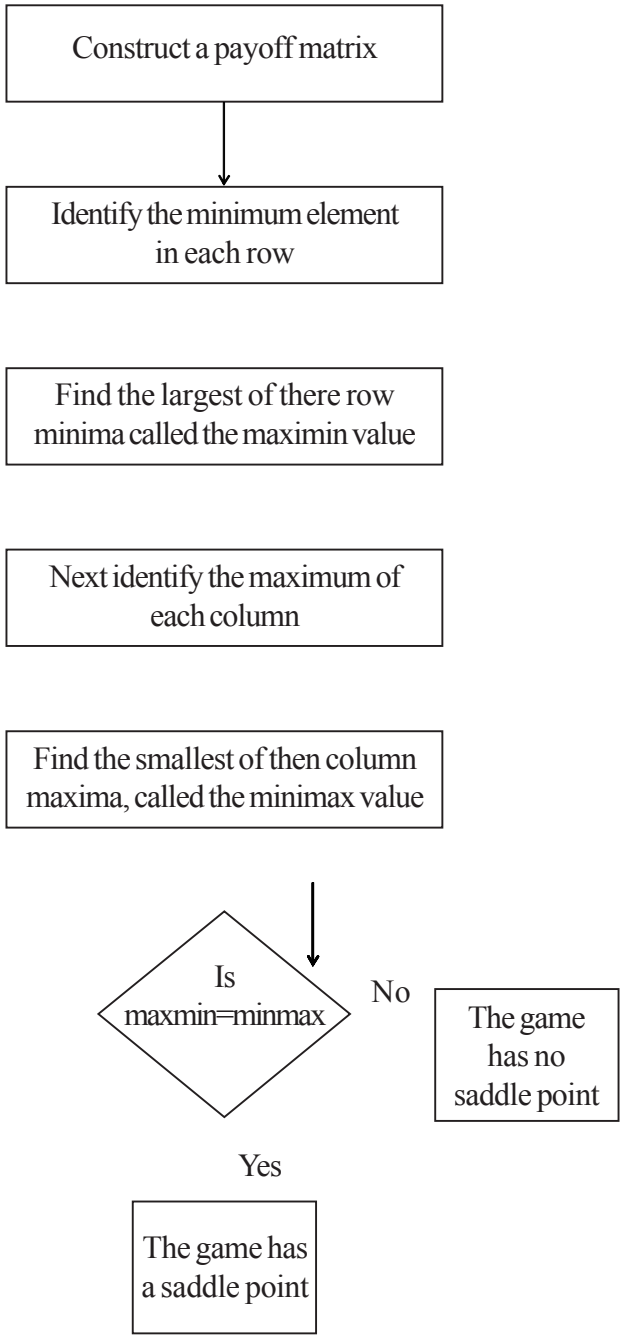
$\Rightarrow$

$\Rightarrow$

Solution of a Rectangular game with saddle point Rupes for determining a saddle point :
Following are the three easy steps to determine the saddle point of a payoff matrix.
1. select the minimum element in each row of the payoff matrix and mark them '○'.
2. select the greatest elements in each column of the payoff matrix and mark them '□'.
3. If there appears an element in the payoff matrix where both '○' and '□' appears, then the position of that element is the saddle point of that payoff matrix.

$\max_j \left[\min_i \{a_{ij}\}\right] = \min_i \left[\max_j \{a_{ij}\}\right] \forall i \text{ and } j \max_i \left[\min_j \{a_{ij}\}\right] \geq a_{ij} \geq \min_{ij} \left[\max_j \{a_{ij}\}\right] \left\{a_{ij}\right\}$

Low Chart to Identify a saddle point



1. Determine which of the following games are strictly determinable and fair. Give optimum strategies for each player in case of strictly determinable games and also find the value of the game.

(a)

$$\begin{array}{c} \\ \text{Player A} \end{array} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix} \end{array}$$

(b)

$$\begin{array}{c} \\ \text{Player A} \end{array} \begin{array}{c} \text{Player B} \\ \begin{bmatrix} 0 & 2 \\ -1 & 4 \end{bmatrix} \end{array}$$

Sol^{n} : (a) The payoff matrix for the player A is,

	B_1	B_2	Row minima ($\underline{\vee}$)
A_1	5	0	0
A_2	0	2	0
column maxima	5	2	
	( )		

First we find the minimum of each row and in circle them. Next we find the maximum of each column and put □ around their. As there is no value where both ○ and □ appears, the game has no saddle point.

In usual notation

∵ , the game is not strictly determinable.

(b) The payoff matrix for the player A is

	B_1	B_2	Row minima ($\underline{\vee}$)
A_1	0	2	0
A_2	–1	4	–1
Column maxima	0	4	

In usual rotation,

$\underline{\vee} = \overline{\vee} = 0$

Therefore, the game is strictly determinable and fair. $\underline{\vee} \neq 0, \overline{\vee} \neq 2$

Optimum strategy for the player A is A_1

Optimum strategy for the player B is B_1

value of the game=0

2. For the game with payoff matrix

	A_1	A_2	A_3	Row minima ($\underline{\vee}$)
A_1	–1	2	–2	–2
A_2	6	4	–6	–6
Column maxima	6	4	–2	
	($\overline{\vee}$)			

In usual notation, =

Hence the game has a saddle point and is strictly determinable.

optimum strategy for the player B is B_1

optimum strategy for the player A is A_3

and the value of the game is given below. Find the solution of the game for players A and B.

		B				
		I	II	III	IV	V
A	I	–2	0	0	5	3
	II	3	2	1	2	2
	III	–4	–3	0	–2	6
	IV	5	3	–4	2	–6

Sol^{n} : The payoff matrix is as given below.

	B					Row minima ( )
	I	II	III	IV	V	
I	(–2)	0	0	[5]	3	–2
II	3	2	[(1)]	2	2	1*
III	(–4)	–3	0	–2	[6]	–4
IV	[5]	[3]	–4	2	(–6)	–6
Column maxima ($\overline{\vee}$)	5	3	1*	5	6	

The game has a saddle point at the position (II, III)

(i) best strategy for the player A is II

(ii) but strategy for the player B is III

(iii) value of the game for A is 1 and for B, it is –1

solve the following game.

	B		
A	6	8	6
	4	12	2

Sol^{n} : To find the saddle point of the above payoff matrix we proceed as discussed earlier.

	B_1	B_2	B_3	Row minima
A_1	[(6)]	8	[(6)]	6
A_2	4	[12]	(2)	2
Column maxima	6	12	6	

Clearly, the above payoff matrix has two saddle points at (1, 1) and (1, 3) positions. Thus the solution of the game is given by,

(i) optimum strategy for the player A is 1.

(ii) optimum strategy for the player B is either 1 or 3.

(iii) values of the game is 6 for A and –6 for B.

5. The payoff matrix of a game is given below. Find the best strategy for each player and also the value of the game. Is this game fair?

	B_1	B_2	B_3	B_4
A_1	–5	2	0	7
A_2	5	6	4	8
A_3	4	0	2	–3

Sol^n : At first we find the saddle point of the following payoff as given below :

	B_1	B_2	B_3	B_4	Row minima
A_1	(–5)	2	0	7	–5
A_2	(5)	6	(4)	8	4
A_3	4	0	2	(–3)	–3
Column maxima	5	6	4	8	

The game has a saddle point at the position (2, 3) $\underline{\nabla}$

(i) best strategy for the player A is A_2.

(ii) best strategy for the player B is B_3.

(iii) value of the game for A is 4 and for B, it is –4.

since $= \underline{\nabla}$ 0,, the game is not fair.

Check your progress

1. Consider the games will the payoff matrix as given belows.

(i)

		Player B	
		B_1	B_2
Player A	A_1	1	2
	A_2	4	–3

(ii)

		Player B	
		B_1	B_2
Player A	A_1	–5	2
	A_2	–7	–4

Determine which of the above games are strictly determinable and fair? Give the optimum strategies for both the players.

2. Solve the following games whose payoff matrix are given below.

(i)

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	–5	3	1	10
	A_2	5	5	4	6
	A_3	4	–2	0	–5

(ii)

		Player B				
		B_1	B_2	B_3	B_4	B_5
Player A	A_1	–2	0	0	5	3
	A_2	3	2	1	2	2
	A_3	–4	–3	0	–2	6
	A_4	5	3	–4	2	6

Rectangular Games Without Saddle Point : Mixed Strategy

As we have learnt till now, a rectangular game with payoff matrix has a saddle point (r, s), then i=r and j=s is the optimal strategy of the game. On the otherhand, where the maximum of the row minima and the minimum of the column maxima are not equal, the maximin minimax (pure strategy) principle of solving a game breaks down. To deal with such a situation, the concept of chance move is introduced. When a player makes his choice from a set of categories not by his own decision, but by some chance mechanism, the move is called the chance move. (a_{ij})

To differentiate between pure and mixed strategies we may cite here the examples of chess and card games. Chees is a game where only pure strategies are involved, whereas, in most of the card games mixed strategies are involved, i.e. pure strategy as well as chance move. The beauty of a mixed strategy lies in teh fact that a player may mislead his opponent into a wrong decision, for his own advantage by behaving unpredictably. Here the opponent have to keep guessing always. In case of mixed strategy both the players make move in a random manner, not caring about his loss in an individual game. ..... aim of each player is to optimize the average payoff over a large number of plays.

Illustrated Examples :

1. In a game of matching coins with two players, suppose a player wins Rs 2 when two heads appear and wins nothing when there are two tails. Looses Rs 1 when one head and one tail appears. Determine the payoff matrix, the optimal strategy for both the players and also the value of the game.

Sol^n : The payoff matrix for the game is as given below :

		Player B		
		B_1	B_2	Row minima
Player A	H	2	–1	–1
	T	–1	0	–1
Column maxima		2	0	

Using Maximin Minimax principle to the above payoff matrix we see that,

Maximin value $(\underline{\vee}) = -1 \neq$ minimax value $(\overline{\wedge}) = 0$ so, the payoff matrix doesnot have a saddle point.

Now we shall see how one player can find best strategy and how to find expected amount to be gained or lost by the player.

Let the optimum mixed strategy for the player A will be to min H and T with probabilities p and 1–p respectively such that p+(1–p)=1

Let the mixed strategy be,

$$\begin{bmatrix} H & T \\ p & 1-p \end{bmatrix}, \quad p+(1-p)=1$$

Now, the problem is to find the value of p.

The expected gain for the player A when B plays mixed strategy H alone is,

E(A, H)=p.2+(1–p) (–1)=3p–1

Similarly, expected gain for the player A when B plays T alone is,

E(A, T)=p(–1)=–p

Since p has been chosen so as to make A's mixture of moves optimum against any two possible moves of B, we must have,

3p–1=–p

$\Rightarrow$ 4p=1

p=

Hence the best strategy for the player A is to play H and T with probability $1/4$ and $3/4$ respectively. Thus this mixed strategy is usually denoted by $\begin{bmatrix} 1 & 3 \\ 4 & 4 \end{bmatrix}$. So the expected gain for the player A is,

$$E(A) = 2.\frac{1}{4} + (-1).\frac{3}{4} = -\frac{1}{4}$$

Now, whatever be the moves of the player B, A's expected you will remain equal to $\left(-1/4\right)$

Similarly, for the player B,

E(B, H)= q.2+(1–q) (–1)=3q–1

E(B, T)=q(–1)+(1–q).0=–q

Equating we get, $q = \frac{1}{4},\ 1-q = \frac{3}{4}$

$$\therefore E(B) = \frac{1}{4}.2 + \frac{3}{4}.(-1) = -\frac{1}{4}$$

Hence the complete solution of the game is,

(i) The player A should play H and T with probabilities $\frac{1}{4}$ and $\frac{3}{4}$ respectively and the optimal strategy for the player A is $x_o = \left(1/4, 3/4\right)$

(ii) Optimal strategy for the player B is, $x_o = \left(\frac{1}{4}, \frac{3}{4}\right)$

(iii) The value of the game is $-\frac{1}{4}$ to the player A. Here (x_o, y_o) is the strategic saddle point of the game.

Check your progress

Find the optimal strategies for the games for which payoff matrius are given below :

(i)

		Player P_2	
		I	II
Player P_1	I	5	1
	II	3	4

(ii)

		Player B	
		I	II
Player P_1	I	1	3
	II	4	2

Important properties of Optimal Mixed Strategy

1. If one of the players sticks to his optimal strategy and the other player deviates from the optimal strategy, then the player who deviates from the optimal strategy can never have greater yield.
2. If a fixed number k is added to each element of the payoff matrix, then the value of the game is increased by k, though the optimal strategy remains unchanged.
3. If every element of a payoff matrix is multipled by a constant k, the value of the game will be k times the value of the original game, the optimal strategy remains unchanged.

Two-by-two (2x2) game without Saddle Point $\frac{x_1}{x_2} = \frac{a_{22} - a_{21}}{(a_{11} + a_{22})}$

Till now we have learnt that to solve a 2×2 game without a saddle point, the best strategy is the mixed strategy. So, we are interested to find out the probabilities with which each of the strategies will be selected. We prove the following theorem for this.

Theorem– For any two person zero game, where optimal strategies are not pure strategies (i.e. there is no saddle point) for which the payoff matrix for the player A is,

		B	
		y_1	y_2
A	x_1	a_{11}	a_{12}
	x_2	a_{21}	a_{22}

the optimal strategies (x_1, x_2) and (y_1, y_2) are determined by,

$$, \quad \frac{y_1}{y_2} = \frac{a_{22} - a_{12}}{(a_{11} - a_{21})}$$

and the values of the game to the player A is,

$$V = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

Proof : Let the mixed strategy for the players A and B be given by, (x_1, x_2) and (y_1, y_2)

$\therefore \quad x_1 + x_2 = 1 \qquad$ —— (1)

and $y_1 + y_2 = 1 \qquad$ —— (2)

obviously, $x_1, x_2, y_1, y_2 \geq 0$

Now the expected gain to A, when B moves his first strategy is,

$a_{11} x_1 + a_{21} x_2$

Similarly, expected gain to A, when B plays his 2nd strategy is,

$a_{12} x_1 + a_{22} x_2$

Again, expected gain to player B, when A plays strategy 1 and 2 respectively are

$a_{11} y_1 + a_{12} y_2$

$a_{21} y_1 + a_{22} y_2$

If v is the value of the game, then A will expect to gain atleast v.

—— (3)

And also B expects to loose at the most v.

i.e. $\left.\begin{aligned} a_{11} y_1 + a_{12} y_2 &\leq v \\ a_{21} y_1 + a_{22} y_2 &\leq v \end{aligned}\right\}$ —— (4)

Now, our problem is to find x_1, x_2, y_1, y_2 satisfying (1), (2), (3) and (4).

For optimum strategies enequalities (3) and (4) becomes equalities.

i.e. $a_{11} x_1 + a_{21} x_2 = v \qquad$ —— (5)

$a_{12} x_1 + a_{22} x_2 = v \qquad$ —— (6)

$a_{11} y_1 + a_{12} y_2 = v \qquad$ —— (7)

$a_{21} y_1 + a_{22} y_2 = v \qquad$ —— (8)

Now subtracting (6) from (5) we get,

$(a_{11} - a_{12})\, x_1 = (a_{22} - a_{21})\, x_2$

$\Rightarrow \qquad$ —— (9)

$\therefore \quad x_2 = \qquad$ —— (10)

From (1) and (9) we get,

$\Rightarrow x_1 = \qquad$ ——(10)

Again subtracting (8) from (7) we get,

—(11)

From (2) and (11) we ge,

$$y_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} \quad \text{—(12)}$$

Subtotelesting the values of x_1, y_1 in any one of the equations (5), (6), (7) and (8) we get,

If ratios $\frac{x_1}{x_2}$ and $\frac{y_1}{y_2}$ are both position, they will give acceptable values of x_1, x_2, y_1, y_2.

Further for all such games, in a payoff matrix, the largest and the second largest element must his in one of the diagnals. Here in 2×2 game without a saddle point only following 8 orders are possible.

$$\left.\begin{aligned} a_{12} \geq a_{21} \geq a_{11} \geq a_{22} \\ a_{12} \geq a_{21} \geq a_{22} \geq a_{11} \end{aligned}\right\}$$

$$\left.\begin{aligned} a_{22} \geq a_{11} \geq a_{21} \geq a_{12} \\ a_{22} \geq a_{11} \geq a_{12} \geq a_{21} \end{aligned}\right\}$$

$$\left.\begin{aligned} a_{21} \geq a_{12} \geq a_{11} \geq a_{22} \\ a_{21} \geq a_{12} \geq a_{22} \geq a_{11} \end{aligned}\right\}$$

$$\left.\begin{aligned} a_{11} \geq a_{22} \geq a_{12} \geq a_{21} \\ a_{11} \geq a_{22} \geq a_{21} \geq a_{12} \end{aligned}\right\}$$

It can be easily proved that with the above orderings the ratios $\frac{x_1}{x_2}$ and $\frac{y_1}{y_2}$ are non-negative.

Note : This theorem is applicable for 2×2 game without a saddle point only.

Illustrated examples :

1. The payoff matrix of a game is given below :

		B	
		I	II
A	I	6	3
	II	4	5

(i) Solve the game.

(ii) Find the value of the game.

Sol^{n} : We at first trye to find the saddle point of the game in usual manner :

and V=

=

=

Hence the optimal strategy for the player A is $\left(\frac{1}{4}, \frac{3}{4}\right)$

the optimal strategy for the player B is $\left(\frac{1}{2}, \frac{1}{2}\right)$

value of the game to A = 4.5

2. Solve the following two person zero sum game.

		B	
		I	II
A	I	10	5
	II	7	8

Solution : In usual way we first try to find the saddle point of the game as shown below :

		B		
		I	II	Row minima
A	I	[10]	(5)	5
	II	(7)	[8]	7
Column maxima		10	8	

Here, maxmin Row = $\underline{\vee} = 7 \neq$ Minmax column $\doteq \overline{\vee} = 8$

Hence the game has no saddle point, and here the optimal strategy would be the mixed strategy.

		B		
		I	II	Row minima
A	I	[6]	(3)	3
	II	(4)	[5]	4
Column maxima		6	5	

Here max min Row $= \underline{\vee} = 4 \neq$ min max column=5

Hence, the game has no saddle point. Hence, the optimal strategy would be a mixed strategy.
Let the player A chooses his I and II strategy with probabities x_1 and x_2 respectively, $x_1+x_2=1$ and the player B chooses his strategy I and II with prob (y_1, y_2) such that $y_1+y_2=1$

$\therefore\ x_1 =$

$=$

$=$

$\therefore\ x_2 =$

11y, $y_2 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

$=$

$\therefore y_2 = \frac{1}{2}$ [illegible] $\frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

At the player A chooses his strategy I and II with probability (x_1, x_2) and the player B chooses his strategy is with probability (y_1, y_2)
such that, $x_1+x_2=1$, $y_1+y_2=1$

Now, $x_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

$=$

$=$

$\therefore\ x_2 =$

and $y_2 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

$=$

=

$\therefore\ y_2 =$

and $v = \dfrac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

=

=

Hence, the optimal strategy for the player A is $\left(\frac{1}{6}, \frac{5}{6}\right)$

optimal strategy for the player B is $\left(\frac{3}{6}, \frac{3}{6}\right)$

and the value of the game for player A is 7.5.

Check your progress

1. For the games with following payoff matrix, determine the optimal strategies and the value of the game.

(i)
$$\begin{array}{c} \\ A \\ \\ \end{array} \begin{array}{c} B \\ \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

(ii)
$$A \begin{array}{c} I \\ II \end{array} \begin{bmatrix} 8 & -3 \\ -3 & 1 \end{bmatrix}$$

Principle of Dominance to reduce the size of the Game :

The rule of dominance is basically used to reduce the size of the payoff matrix. Sometimes in a rectangular game, situations arise when same strategies dominate others. This happens when one or more pure strategies of a player is superior to atleast one remaining strategy of the payoff matrix, such that the inferior strategy is never used. We say that the inferior pure strategy is dominated by the superior pure strategy. In such case, we can reduce the size of the payoff matrix by remaining the strategies which are dominated by other strategies. The role of dominance is used specially in case of two person zero sum game without saddle point.

Dominance rule is applied as stated below :

(i) If all the elements of the i^{th} row of the payoff matrix are less than or equal to the corresponding element of the, say, j^{th} row, then the i^{th} strategy is dominated by the j^{th} strategy. In this case, the player A never uses i^{th} strategy.

(ii) If all the elements of the r^{th} column are greater than or equal to the corresponding element of the, say,

r^{th} column, then the r^{th} strategy is dominated by the s^{th} strategy. In this case the player B will never choose the r^{th} strategy.
(iii) A pure strategy may also be dominated if it is inferior to the average of two or more pure strategies.

Note : Using dominance property, we always try to reduce the size of the payoff matrix to a 2×2 matrix, so that the game can be solved by using the methods discussed so far.

Illustrated Examples :
1. The payoff matrix of a competetive situation is as below.

(i) Reduce the size of the game to a 2× matrix by the method of dominance.
(ii) Determine the optimal strategies.
(iii) Find the value of the game.

Sol^{n} : At first we proceed to find the saddle point of the game by maximin minimax principle as show below.

		B			
		I	II	III	Row minima
A	I	–1	(–2)	+6	–2
	II	7	[5]	(–1)	–1
	II	[6]	0	[12]	0

Column maxima 6 5 12

$\neq$

		B		
		I	II	III
A	I	–1	–2	6
	II	7	5	–1
	III	6	0	12

as here maxmin = 0 minmax value = 5, the game doesnot have a saddle point. Hence we try to reduce the size of the matrix by the use of dominance rule. Here since every element of the 1st row of the payoff matrix is smaller than the corresponding elements of the 3rd row, from A's point of view, strategy I is dominated by the strategy III. So our reduced matrix becomes,

		B		
		I	II	III
A	I	7	5	–1
	II	6	0	12

Again every element of column I is greater than the corresponding elements of column II, by dominance

rule (2), from player B's point of view, strategy II dominates strategy I and accordingly the payoff matrix reduces to a 2 2 matrix.

		A	
		II	III
B	II	5	–1
	III	0	12

Examing we see that the game has no saddle point and also it is not reducable any further. Hence the optimal strategies will be the mixed strategies.

Lte us assume that the player A chooses his mixed strategy with probabilities (x_1, x_2) and the player B chooses his mixed strategies with probabilities (y_1, y_2) respectively.

such that, $x_1+x_2=1$

$y_1+y_2=1$

Now, $x_1 =$

$x_2 =$

11_y, $y_1 = \dfrac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \dfrac{13}{18}$

$\dfrac{5 \quad 22 \quad a_{11}-a_{21}}{10, \quad 33 \quad a_{22}-(a_{12}+a_{21})} = \dfrac{2}{3}$

$y_2 =$

and $v = \dfrac{a_{11}a_{22}-a_{12}a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \dfrac{10}{3}$

A's optimal strategy is

B's optimal strategy is $\left(0, \dfrac{13}{18}, \dfrac{5}{18}\right)$

and the value of the game is $\dfrac{10}{3}$ for the player A.

2. Player A and B plays a game in which each his three coins a 5p, 10p and a 20p. Each select a coin without the knowledge of the other's choice. If the sum of the coins is an odd amount, then A wins B's coin. But if the sum is even, then B wins A's coin. Find the best strategy for each player and the value of the game.

Sol^n : The payoff matrix for the player A is,

		Player B		
		B_1 5p	B_2 10p	B_3 20p
	A_1 5p	–5	10	20
Player A	A_2 10p	5	–10	–10
	A_3 20p	5	–20	–20

Sol^n : Obviously, the game has no saddle point, so we try to reduce the size of the given payoff matrix by dominance rule. Now every element of column B_3 is more than or equal to the corresponding element of row B_2. Thus the strategy B_3 is inferior to B_2. So the reduced matrix becomes

	B_1	B_2
A_1	–5	10
A_2	5	–10
A_3	5	–20

Now from A's point of view, it is seen that the player A will never use his strategy A_3, which is dominated by the strategy A_2. Hence the size of the matrix can further be reduced as follows :

	B_1	B_2
A_1	–5	10
A_2	5	–10

Thus we get 2×2 payoff matrix, where maximin value is not equal to the minimax value. Hence the game has no saddle point and we solve it by applying the concept of mixed strategy.

Let us assume that the players A and B chooses their optimal strategies with the probabilities (x_1, x_2) and (y_1, y_2) respectively, such that, $x_1+x_2= 1$, $y_1+y_2= 1$

$x_1 =$

$x_2 =$,

$$y_1 = \frac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})} = \frac{-10-10}{(-5-10)-(10+5)} = \frac{2}{3}$$

$y_2 =$

and $v = \dfrac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \dfrac{50 - 50}{(-5 - 10) - (10 + 5)} = 0$

Hence, A's optimal strategy is

B's optimal strategy is $\left(\frac{2}{3}, \frac{1}{3}, 0\right)$

value of the game is 0 to player A.

Arithmatic Method or the Method of Oddments for the solution of a2×2 game without saddle point :

Arithmatic method may be used to find the optimal strategies for each player of a 2 2 game without saddle point. The method consists of the following steps.

1. Find the difference between the two values in column I and put it under column II, neglecting –is sign if any.
2. Find the difference between the two values in column II and put it under column I, neglecting –is sing as before.
3. Repeat the procedeur for the rows as well.

These values are called oddments and represents the frequency with which the players use their courses of action. These frequencies are concerted to probabilities by deviding them by the sum of frequencies for each player. We illustrate the method with the help of an example.

1. Solve the following game by the method of oddments

		A	
		II	III
A	I	0	11
	II	3	4

$\left(\frac{3}{5}, \frac{1}{2}, 0\right)$

Sol^{n} : Since the game has no saddle point, the optimal strategy will be a mixed strategy.

(1) We find the difference between the values of the first column i.e. 5–3=2 and put it under column II.

(2) Next we find the difference between the elements of column II, i.e. 1–4=3 (neglecting sign) and write it under column I.

These are the oddments with which the player A choose his courses of action.

(3) Next we find the probabilities by deviding 2 by (2+3)=5 and also 3 by 5, These are the probabilities with which the player A choose his courses of action.

(4) Similarly we find the probabilities for the playe B, i.e. and $\frac{2}{5}$, for selecting courses of action I and II respectively. The results are displayed below :

		B			
		I	II	oddments	probabilities
A	I	5	1	1	$\frac{1}{5}$
	II	3	4	4	$\frac{4}{5}$

oddments	3	2
probabilities	$\frac{3}{5}$	$\frac{2}{5}$

To find the value of the game using A's oddments

If B chooses strategy I, the value of the game

$$v = 5.\frac{1}{5} + 3.\frac{4}{5} = \frac{17}{5}$$

If B chooses strategy II, the value of the game is,

$$v = 1.\frac{1}{5} + 4.\frac{4}{5} = \frac{17}{5}$$

Similarly using B's oddments,

$$v = 5.\frac{3}{5} + 1.\frac{2}{5} = \frac{17}{5}$$

and $v = 3.\frac{3}{5} + 4.\frac{2}{5} = \frac{17}{5}$

Hence the solution of the game is,

$$A\left(1/5, 4/5\right),\ B\left(1/5, 4/5\right),\ v = 17/5 \text{ to A}$$

Check your progress

Minitations of Game Theory

Though Game theory was initially received with much enthusiasm as an insight into logical reasoning for selection of optimal strategies by the players, it has got some serious limitations in it's use to solve situations. They are as discussed below :

1. The very assumption that the players have the knowledge of his own payoff as well as his opponents, is rather unrealistic. The players may just guess of his own and his reval's payoff.
2. The analysis of the game strategy becomes more and more complex, as the number of players goes on increasing.
3. The competitors in real life situations are neither of equal importance nor they have equal knowledge.
4. In zero sum games, it is assumed that the gain to one player is equal to the loss to the other players. But in real life situation both the parties may gain at the same time.

UNIT 4

Learning Objectives :

After going through this chapter students will be able to understand.

(i) the situation in which queuing problems are generated.
(ii) the meaning and objectives of a queuing model.
(iii) the basic characteristics of a queeing model, including the queue discipline.
(iv) customer behaviour in a queue.
(v) various objectives that may be set for the operation of a waiting time.

Queuing Theory

Introduction : In our day to day life we all face a very common situation– queuing or waiting in a line. These queues form ticket booth, doctor's clinic, bus stoppage and so on. Queus are also observed in a repair shop where machines are waiting to be repaired, in a warehouse where items wait to be used or in a telephone exchange where incoming calls are in queue to be answered.

Whereour people arrive at a cinema hall window in large numbers, they well obviously will have to wait for getting their tickets. In order to maintain proper discipline, there is no other alternative but to form a queue; which is called the waiting line. On the otherhand, the person issuing ticket have to wait, i.e. remain idle untill people arrive. Here, people arriving for tickets are called the customers and the person issuing ticket is termed as server. As discussed earlier, a customer or servers need not necessarily be human beings, but may be of aeroplanes seeking to land at a busy airport, ships waiting to be unloaded, jobs waiting to be processed by a computer etc.

Thus in any queuing situation, there exists two cases :

(i) someone or something that requires service, which we usually referred to as customers.
(ii) Someone or something that completes service as required by customers, usually referred to as servers.

So what we have discussed so far is that, in general, queues are formed where the units that require service, referred to as customers, wait for service in the service facility or the servers stand idle and wait for customers. Customers have to wait where the number of units waiting exceeds the number of service facilities and the servers have to wait when the number of service facilities exceeds the number of waiting customers. However, the arriving customers or arriving units may form one line and may be served through onle one server, they may form one line and may be served through several servers or they may form several lines and may be served through several servers. Queuing system of first type is observed in doctor's chamber, second type of queuing system may be found in barbar's shop and the third type may be observed in the billing counter of a super market.

Moreover, in case, there is more than one server, they may be in parallel or in series set-up. In case of parallel service channels, the customers upon arrival form a single queue in front of each of the servers. In case the service facility in series set-up, the customers must pass through the service facility one after another. Thus queuing theory is the planning and analysis of the service capacity. If gives us the probability distribution of the number present in the queue, from which we can find the mean and variances of queue length and the probability distribution of the waiting time of a customer or the probability distribution of servers busy period.

Queuing theory owes it's development to Danish Mathematician and engineer A. K. Exlang. While working in the copenhoque telephone exchange, be became interested to analyse and optimize it's operations. He wanted to minimize on-hold time of people (i.e. a telephone queue) and was deter-

mined to find out how many circuits were needed to provide fairly good level of telephone service. He was curious to find out how many telephone operators were needed to process a given level of calls. He published his paper on 'Telephone waiting times' in the year 1920, which is considered as first of it's find where some of the first queuing models were discussed. Thus Erlang is the pioneer in the field of applied queuing theory. Thus many practical situation can be put in the queuing framework. Some examples are provided in the following table :

Table : Some Queuing problem examples

Problem	Customer	Services facility
Determining the number of clerks required at the checkout of a super marker	Shoppers	Clerks
Scheduling of patients in a clinic	Patient	Doctors
Determining the size of a parking lot	Vehicles	Parking space
Determination of number of runways at the airport	Aeroplane	Runways
Number of circuits required in a Telephone exchange	Telephone calls	Circuits
Number of Repair persons in a Machine repair shop	Broken machines	Repair person
Determining the numf cabs for a flut	Public	Cabs

Serving and objective of a Queuing Model :

Queuing model may be defined as a model used to represent a service oriented problem, where customers services randomely to receive some service. Queuing models are very helpful in determining how to operate a queuing system, the service time being also a random variable. A queuing model is constructed so that the queue length and waiting time can be predicted. These models are typically important in business and software applications.

The main objective of queuing model is essentially to minimize the total costs. Generally, there are two categories of costs associated with a queuing model.

(i) costs associated with the customers waiting for service,

(ii) costs associated with capacity.

The first type of cost may include salaries to be paid to an employee while he waits for his service, such as drivers of trucks waiting to unload or machine waiting for parts or tools. It also includes cost of waiting space for the customers.

–Again capacity costs are the costs incurred in maintaining the ability to provide service. For example, costs in maintaining the checkout counters in a super market.

Thus a traditional queuing model aims to create a balance between the cost of providing a good level of service and the cost of customers waiting for services. Thus the main reason behind the need of analysis of a queuing model is that the customers always regard waiting as a non-value added acitivity. Again, the managers have all the reason to be concerned with the waiting lines. The main concern of the managers are :

(i) the cost of providing waiting space,

(ii) the possible risk of loss in business, in case the customers refuse to wait or leave the waiting line, without beeing served, if the waiting time is too long,

(iii) risk of customer dissatisfaction,

(iv) possible loss of good will, and

(v) the congestion in one area of business may desrupt the other area of business.

Application of Queuing Theory :

Queuing theory is useful because of it's countless and diverse application. Real life application of queuing theory cover a wide range of businesses. It's findings may be used to provide faster customer serivice, increase traffic flow, increase order shipments from a warehouse etc. Following are the some of the applications of queuing theory that are extensively used in our day to day life.

Health Care :

In health care, queuing models are generally based on three factors and the variation within those factors. They are patient arrival rate, server rate (serice time for tests, treatments etc.) and the number of servers (clinical and non-clinical staff) available : Queuing models can help forecast answers to questions about patient flow, such as :

- how long will the average patient have to wait?
- how many staff are necessary for all patients to be examined within a stipulated time? etc.

Business :

By applying queuing theory, a business can develop more efficient systems, processes, pricing mechanism, staffing solution and arrival management stratigees to reduce customer wait times and increase the number of customers that can be served.

Banking :

Banking is one of the most important utility services of the public. Most of the banks use standard queuing models. It is very useful to avaid standing in a queue for a long time and to give service to all customers. Bank is an example of unlimited queue length. Queuing is used to generate a sequence of customers arrival time and to choose randomly between different services : opening an account, transaction with different period of time for each service.

Bank ATMs :

In ATM, bank customers arrive randomly and the service time, i.e. the time customers take to do transaction in ATM, is also random. We can use queuing model to derive the arrival rate, service rate, utilization rate, waiting time in the queue and the average number of customers in the queue. Queuing can help bank ATM to increase its quality service, by anticipating, if there are more customers in the queue.

Traffice System :

The vehicular traffic flow can be maintained and congestion can be minimized by using queuing theory in order to reduce the delay on the roads. A basic model of vehicular traffic based on queuing theory can determine the best times of the red, amber and green lights to be either on or off in order to reduce traffic congestion on the roads. Thus queuing also helps to reduce fuel consumption.

Railway Station :

In the country like India. Where Railway is one of the most popular and cheapest means of transportation, it is always difficult to book confirmed tickets for the journey. The population doesnot match up with number of trains running various routes, espicially those connecting the metro cities. Indian Railways is trying to meet the ever increasing demand of this huge population. The queuing system is used to avaid the inconvenience of passangers and it is feasible and the results are effective and practical.

Computer System :

Many jobs arrive sequentially at a computer system and the execution time is a random variable. Jobs are executed in order of arrival and if the computer is busy when a job arrives, the job is placed in a queue. In the terminology of queuing theory, the computer is the 'server' and the jobs are the 'customers'. The logical structure of a queuing model can determine effectively the appropriate number of computer and operators required for an effective system with minimum queue length.

Logistics :

Queuing theory is useful in guiding the logistics of many business. The operation department of a delivery company, may use queuing theory to help if smooth out the kinks in it's system for moving packages from a warehouse to a customer. In this care queue beeing comprised of boxes do waiting to be delivered to customers.

Queuing system and it's characteristics :

Till now we have learnt that the basic queuing system can be described as a process where customers arrive at a service centre, wait for some time in a queue if the server or the servers are busy servicing others customers and gets served when the server is free and leave the system as soon as the service is complete. A queuing system can be completely described by the following basic characteristics.

(i) The input of the arrival pattern
(ii) Queuing process
(iii) Queue discipline
(iv) Service process or service mechanism

(i) **The input or arrival patern :**

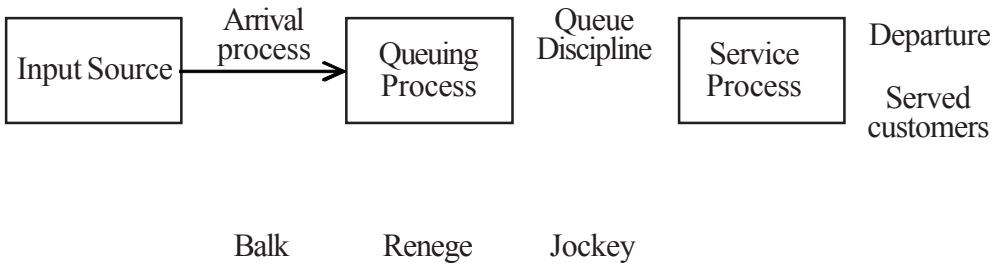
The input describe the way the customers arrive and join the system. The input source or arrival may not be homogeneous population but may be consists of serveral non-homogeneous sub-populations. For example we may consider here is the case of patients arriving at the OPD of a hospital. What we may experiences is that, there are patients with appointments, walk in patients and emergency patients. Each type of patient have different service expectations and have different waiting pattern.

The group of arriving customers may be finite or infinite. Again the behaviour of arriving customers may differ from one another. There are some patient customers, who on arriving the serving station stays in the system until being served, no matter how long he has to wait. Whereas, there are customers who waits in the queue for a certain time and leaves the system after some time, without being served, because of some reasons, such as long queue in front to line. Such an impatient customer is said to be renaged.

Again, there are some customers who gets discouraged by the size of the queue, even before joining the queue. He balks estimating the excussive waiting time for the desired service. Sometimes

customers move from one queue to another, in the hope of receiving service more quickly and are said to be jockeying. Such behaviour is after observed at the checkout points in a supermarket.

In many practical situations the arrival time distribution can be approximated by poison distribution and the inter arrival time is approximated by an expossential distribution. This will be discussed in the next unit in a lucid manner.



Queuing System

2. Queuing Process : The queuing process mainly deals with the number of queues and their respectives lengths. The number of queues mainly depend on the layout of the service system. There may be a single queue or multiple queues or no formal queue at all. Also, the length of the queue depends upon certain factors like waiting space, attitude of customers etc.

A queue may be either a finite or infinite source queue. In case of finite queue, the service station can not accomodate more than a certain number of customers. Those over a certain numbers are not allowed to enter the system until there is space available to eater to the new customers. Such type of finite source queues are observed in cinema halls, restaurants etc. Again, in case of infinite source queue, the service system is free to accomodate any number of customers. Such situations may be observed in a rates counter, where there is no restriction on the number of orders placed. so, in this case a queue of any size can be formed.

Some times an arriving customer experiences a long queue in front of the service station, not even enter the services station, even though the waiting space is available. That is, in such cases the queue length depends on the attitude of the customer. Such situations are observed in petrol stations, when a motorist prefers to seek service in another station, when many vehicles are seen waiting in the service station.

It is to be noted that, in some of the finite source queue system, no queue is allowed to form, such as in the case of parking space (service facility) which cannot accomodate additional incoming vehicles (customers), vehicles are diverted elsewhere.

Again a multiple queue may also be finite or infinite, but it has got serveral advantages like options for the customers to join any queue or can switch queue. As a result, balking behaviour of customers can be controlled to some extent.

3. Queue Discipline : The queue discipline is the manner in which customer forming queues are selected for service. Following queue disciplines are observed as a common practice.

(i) **First Come First Served (FCFS) :** According to this discipline, customers are served in the order of their arrival. This type of queue discipline is most common practice and can be observed in a Railway ticket counter, cinema hall, ticket count or a prepaid taxi counter. In the next unit we shall deal

with queuing model with FCFS queue discipline.

(ii) **Last Come First Served (LCFS) :** Other discipline, which is in common use is last-come first served. This discipline is practiced in big godowns where items that come last are taken out first. This is also practiced in most cargo handling situations where the last item loaded is removed first.

(iii) **Service In Random Order (SIRO) :** Under this rule customers are selected for service at random irrespective of their arrival time in the system.

(iv) **Service in priority basis :** Under this rule customers are grouped in priority classes on the basis of some attributes and FCFS is used within each group to provide services.

Service Process of Service Mechanism :

The service process is concerned with the manner in which customers are served and leave the system. The service mrchanism refers to

(i) **The pattern according to which the customers are served :** If service times are randomly distributed. we have to find out the probability distribution, which can describe the behaviour well. Here, in the next chapter we shall deal with queuing model in which service time distribution follows exponential distribution.

Service Channels : Service channel available to customers may be classified in two categories.

(a) **Single Channel :** Here the arriving customers may form one line and get servived one by one. That is, only one customer can be served at a time. e.g. cinema ticket window.

(b) **Multi-Channel :** In this case, the system may have a number of service channel, which may be arranged in parallel or in series or a combination of both. In case of parallel service channel, several customers can be served simultaneously, whereas, in case of series service channels, a customer must pass through all the channels, before service is complete.

Exercises :

1. What do you understand by a queues? Give some important applications of queuing theory.
2. Write an essay on various characteristics of a queuing system.
3. Describe the fundamental components of a queuing process and give suitable examples.
4. What is queuing theory? In which type of situations it can be applied successfully? Discuss giving examples.
5. Discuss essential features of queuing system.
6. What do you understand by queue discipline and service process?

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Block 3 : Unit 5

Learning objectives :

After going through this unit, students will be able to

(i) understand the distribution of arrival and service true in a queeing system.

(ii) evaluate the distribution of Inter arrival time in a queeing system.

(iii) understand the relationship between poison process and exponential distribution.

(iv) have a good knowledge about (MIMII) : (D/FIFO) model, the steady state solutions, characterisics of the model, waiting time distribution etc.

Availabilit Distribution in Quees : The arrival pattern of customers in a queeing system varies from one system to another. However, there is still a common pattern of arrival of customers, which we may say as "completely random arrival'. In particular, what we see is that arrival time distribution can be approximated by the poisson distribution, whereas the interval between successive arrivals follow negative exponential distribution.

Distribution of arrival : Poisson Process

In any queing theory, distribution of arrival rate and service rate follows poisson process. Assuming that we have the arrival process $\{N(t), t \geq 0\}$, where N(t) denotes the number of arrivals upto time t and N(0)=0. In this section we shall derive the distribution of arrival in a queue, for which we shall have to make following three assumptions (called the axeim) :

Axeim 1 : The number of arrivals in non-overlapping intervals are independent, i.e, the process has independent increments.

Axeim 2 : Probability of exactly one arrival in the interval $(t, t+\Delta t)$ is equal to $\lambda \Delta t + 0(\Delta t)$, where Δt is very small,

where λ is the arrival rate, independent of time t, and $0(\Delta t)$ is such that

$$\lim_{\Delta t \to 0} \frac{0(\Delta t)}{\Delta t} = 0$$

Axeim 3 : The probability of more then one arrival in the interval $(t, t+\Delta t)$ is $0(\Delta t)$. That is probability of two more arrivals in the small time interval Δt is negligible.

i.e.

Theorem : Let n be the rendom variable representing the number of arrivals in some time interval of length t, then n follows poison distribution with parameter λt.

o.e.

Proof : Here we need to find the distribution of $P_n(t)$, where $P_n(t)$ denote the probability of n arrivals in a time interval of length t. Obviously, $n \geq 0$. First, we have to develope differential difference equations for the process in two different situations,

$n>0$ and $n=0$

Let $P_n(t+\Delta t)$ denote the probability of n arrivals in the time interval $(t, t+\Delta t)$. This may happen in any one of the following ways, which are mutually exclusive.

Hence for $n \geq 0$,

$P_n(t+\Delta t)$ = P [n arrival in (0, t) and no arrival in (t, t+ t)]
+P [(x–1) arrival in (0, t) and one arrival in (t, t+ t)]
+P [n–2) arrival in (0, t) and two arrival in (t, t+ t)]
+....+P [0 arrival in (0, t) and n arrival in (t, t+ t)]

Following the arioms (2) and (3), we get the following differential differences equations,

$$= P_n(t)[1 - P_1(t) - 0(\Delta t)] + P_{n-1}(t)[\lambda\Delta t + 0(\Delta t)] + 0(\Delta t)$$

$\Rightarrow$ —— (1)

Providing both sides by Δt and taking limit a t 0 we get,

—— (2)

Now, when n=0, equation (1) reduces to,

$$P_0(t + \Delta t) = P_0(t)[1 - \lambda\Delta t - 0(\Delta t)]$$

$\Rightarrow$

Deviding both sides by Δt and taking limit as t 0, we get,

—— (3)

So we have the differential difference equations,

$$\left.\begin{aligned} \frac{d}{dt}P_0(t) &= -\lambda P_0(t) && , n = 0 \\ \frac{d}{dt}P_n(t) &= -\lambda P_n(t) + \lambda P_{n-1}(t), && n \geq 1 \end{aligned}\right\} \quad \text{——(4)}$$

Now to solve these (n+1) equations we make use of generating function

$$\phi(x, t) = \sum_{n=0}^{\alpha} P_n(t)z^n \quad \text{——(5)}$$

in the unit circle $|z| \leq |$.

Now multiplying the (n+1) differential difference equations given in (4) by $z^0, z, z^2, -----, z^n$ respectively and summing our n from 0 to

$\Rightarrow$

$\Rightarrow$

$\therefore$ —— (6)

Where, c is an orbitrary constant.

Now, to find the value of e we use the initial conditon,

$$\phi(z,0) = 1$$

c=1

Hence equation (6) reduces to —— (7)

Now, from equation (5) we have,

$$\phi(z,t) = P_0(t) + zP_1(t) + z^2P_2(t) + z^3P_3(t) + ...$$

$\therefore$

——(8)

Hence using equation (7) we get,

$$P_0(t) = e^{-\lambda t},$$

$$P_1(t) = \lambda t e^{-\lambda t}$$

$$P_2(t) = \frac{1}{2!}(\lambda t)^2 e^{-\lambda t}$$

.....

$$P_n(t) = \frac{1}{n!}(\lambda t)^n e^{-\lambda t}$$

$$\left.\begin{array}{l} \phi(z,t) = e^{\lambda(z-1)t} \\ \frac{d}{dz}\phi(z,t)]_{z=0} = P_1(t) \\ \frac{d^2}{dz^2}\phi(z,t)]_{z=0} = 2.P_2(t) \\ \frac{d^3}{dz^3}\phi(z,t)]_{z=0} = 3.2.P_3(t) \\ ..... \\ \frac{d^n}{dz^n}\phi(z,t)]_{z=0} = n!\,P_n(t) \end{array}\right\}$$

Therefore, in general we have,

$$P_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!} \text{ for } n \geq 0 \quad \text{—— (9)}$$

which is the poisson probability law with main λt. Thus the prabability of n arrivals in time 't' follows poisson probability law given by (9).

Exponential Distribution : Distribution of Inter arrival time

Theorem : If the arrival pattern in a queing problem follows poisson process, then the inter arrival times T follows exponential distribution and vice versa.

Proof : Let the random variable T represents the inter arrival time, i.e. the time between successive arrivals, then

P(T>t) = P {there is no arrival in time t}

$= P_0(t)$

=

$\therefore$

But, $P(T \leq t)$ is the distribution function of T and let it be denoted by F(t).

F(t)=1–

Hence, the density function f(t) is given by,

$$f(t) = \frac{d}{dt} F(t) = \lambda e^{-\lambda t} \qquad ,0 \leq t \leq \alpha$$

$$= 0 \qquad , t<0$$

Which is the p.d.f. of exponential distribution with parameter λ. (or mean)

Thus we see that, if the mean arrival rate is λ, then the mean time between arrivals is . In a similar fashion, the converse of the above theorem can be proved.

Distribution of Service Time :

In this process we assume that there are n customers in the system at time t=0 and also assume that no arrival occur in the system. Suppose, departures occur at the rate of μ per minute.

..... in the distribution of arrivals, we make the following true axioms :

(i) The number of departures in non-overlapping intervals are independent.

(ii) P_r [one departure during time t]= t+0(t)

(iii) P_r [more then one departure during t]=0(t)^2

Then it can be early shown that the time (t) required to complete the service of a customer follows exponential distribution given by,

$$F(t) = P(T \leq t) = 1 - e^{-\mu t} \qquad \text{and} \qquad F(t) = P(T \leq t) = 1 - e^{-\lambda t}$$

$$= 0 , \qquad t<0$$

and the service time T follows Poisson distribution given by,

P_r[n service in time T]

$$= \frac{e^{-\mu t}(\mu t)^n}{n!}$$

Check your progress

1. Show that the number of arrivals in a queue in time t follows the Poisson distribution, stating the assumption clearly.
2. State the assumptions under which an arrival process is a Poisson process :
3. It the number of arrivals in some time interval follows a Poisson distribution, show that the distribution of the time interval between two consecutive arrivals is exponential.
4. State the three axioms underlying exponential process.

The M/M/I Queing System

This is a queing model, where, both arrivals and departure occur randomly over time. Suppose a system is in state E_n. Now, if an arrival occurs, the system will be in state E_{n+1}. That is, an arrival may thus be considered as birth and a departure may be looked upon as a death. This type of process is referred to as birth-death process.

The Model (M/M/I) : (α /FIFO)

This ia a queing model with poisson arrival, Exponential service single servicing facility and there is no limit on the system capacity. The services discipline is 'First in, First out' basis.

Let $P_n(t)$ be the probability that there are n customers in the system at time t. Now, to write the difference equations for $P_n(t)$, we have to consider how the system can get to state E_x at time (t+ t). Since arrivals and service both independent of each other, then for n 1, it can open in any one of the three mutually exclussive ways, which are,

(i) n units in the system at time t with probability $P_n(t)$
no arrival in the interval (t+ t) with probability [1– t+0(t)]
no service in the interval (t+ t) with probability [1– t+0(t)]

(ii) (n–1) units in the system at time t with probability (t)
one arrival in the interval (t, t+ Δ t) with probability [t+0(t)]
no arrival in the interval (t,t+ t) with probability [1– t+0(t)]

(iii) (n+1) units in the system at time t with probability $P_{n+1}(t)$
no arrival in the interval (t, t+ t) with probability [i– t+0(t)]
one service in the interval (t, t+ t) with probability [t+0(t)]

Hence the probabilities for the three cases are

(i) $P_n(t)$[1– t+0(t)][1– t+0(t)]

(ii) $P_{n-1}(t)$[t+0(t)][1– t+0(t)]

(iii) $P_{n+1}(t)$[1– t+0(t)][t+0(t)]

Now as said earlier, all the three cases are mutually exclusive, then, $P_n(t+ t)$, i.e. the probability that there are n units in the system at time (t+ t) can be obtained by adding the probabilities for all the above three cases.

$$P_n(t+ t) = P_n(t)[1– t+0(t)][1– t+0(t)] + P_{n-1}(t)[t+0(t)][1– t+0(t)] + P_{n+1}(t)[1– t+0(t)][t+0(t)]$$

Now combining all 0(t) terms and neglecting 0(t)^2, be above equation becomes,

$$P_n(t+ t)=P_n(t)[1–(+) t]+P_{n-1}(t)[t] + P_{n+1}(t)[t]+0(t), \quad n \quad 1$$

$$P_n(t+ t)–P_n(t)= –(+) t P_n(t)+ t P_{n+1}(t) + t P_{n-1}(t)+0(t) \qquad \text{—— (1)}$$

Similarly, no unit in the system at time (t+ t) may happen in following two nutually exclusive ways i.e. (i) no unit in the system at time t with probability $P_0(t)$
no arrival in the system in the interval (t+ t) with probability [1– t+0(t)]
the probability in this case is $P_0(t)$[1– t+0(t)]

(ii) One unit in the system at time t with probability $P_1(t)$
no arrival in the interval (t, t+ t) with probability [1– t+0(t)]
one service in the interval (t, t+ t) with probability [t+0(t)]
Probability for this case is $P_1(t)$[1– t+0(t)][t+0(t)]

Hence as in the precious case,

$P_0(t+\ \ t)=P_0(t)[1-\ \ \ t+0(\ \ t)]+P_1(t)[1-\ \ \ t+0(\ \ t)\ [\ \ \ t+0(\ \ t)]$

..... assumptions as in the case for n 1, we get,

$P_0(t+\ \ t)-P_0(t)=-\ \ \ t\,P.(t)+\ \ \ t\,P_1(t)+0(\ \ t)$ —— (2)

Now, deviding both the equations (1) and (2) by t and taking limit as t 0, we get,

—— (3)

These equations are called the differential difference equations. To get steady state solution for P_n, the probability of n customers in the system at an orbitrary point of time, we take limit as $1 \rightarrow$ in equation (3)

Now, when the system reaches the steady state,

Hence, under steady state system, the equation (3) reduces to

$$\left.\begin{aligned} -(\lambda+\mu)P_n+\lambda P_{n-1}+\mu P_{n+1} &= 0 \\ -\lambda P_0+\mu P_1 &= 0 \end{aligned}\right\} \quad \text{—— (4)}$$

$$\left.\begin{aligned} \lim_{t\to\alpha}\frac{P_n(t)-P_n}{t} &= -(\lambda+\mu)P_n(t)+\mu P_{n+1}(t)+\lambda P_{n-1}(t),\ n\geq 1 \\ P_0'(t)=\frac{d}{dt}P_0(t) &= -\lambda P_0(t)+\mu P_1(t), \qquad n=0 \end{aligned}\right\}$$

Note : Steady state solution exists only when $\lambda<\mu$ where .

If , there is no queue.

If , the state is called the explosive state.

Solution of steady state equations

From 2nd equation of (4) we get,

,

Where $\frac{\lambda}{\mu}=P<1$ is called the traffic intensity.

From the first equation of (4), putting n=1, we get,

$$\Rightarrow P_2=\left(\frac{\lambda}{\mu}+1\right)P_1-\frac{\lambda}{\mu}P_0$$

$$=\frac{\lambda}{\mu}P_1+P_1-\frac{\lambda}{\mu}P_0$$

$=$

$= \left(\frac{\lambda}{\mu}\right)^2 P_0$

$= \rho^2 P_0$

Again putting n=2 in the first equation of (4) we get,

$$P_3 = -\frac{\lambda}{\mu}P_1 + \left(\frac{\lambda}{\mu}+1\right)P_2$$

$$= -\frac{\lambda}{\mu}P_1 + \frac{\lambda}{\mu}P_2 + P_2$$

$$= \left(\frac{\lambda}{\mu}\right)^3 P_0$$

$$= \rho^3 P_0$$

Proceeding in this way we get,

$P_n = \rho^n P_0$ ..... , $n \geq 0$

But, $\sum_{n=0}^{\infty} P_n = 1 \Rightarrow P_0 + P_1 + P_2 + ... + P_n = 1$

$$\Rightarrow P_0(1 + \rho + \rho^2 + ... + \rho^n) = 1$$

$$\Rightarrow P_0(1-\rho)^{-1} = 1$$

$$\Rightarrow P_0 = 1 - \rho$$

$\therefore P_n =$

Which is the probability that there are n units in the system at any time.

Characteristics of the Model (M/M/I) : (α IFIFO)

(i) To find the probability that the queue size is greater than or equal to n, the number of customers.

(queue size $\geq$ n) =

$$= \sum_{k=n}^{\alpha} (1-\rho)\rho^K$$

$$= (1-\rho)\sum_{k=n}^{\alpha} \rho^K$$

$$= (1-\rho)[\rho^n + \rho^{n+1} + ...]$$

$$= (1-\int)\int^n (1+\int+\int^2+.....)$$

$$= (1-\int)\int^n (1-\int)^{-1}$$

$$= \int^n$$

(ii) **To find the average number of customers in the system (expected queue length)**

$$E(n) = \sum_{n=0}^{\alpha} P^n = \sum_{n=0}^{\alpha} n(1-\int)\int^n$$

$$= (1-\int)\sum_{n=0}^{\alpha} n\int^n$$

$$= (1-\int)\,[\int+\theta\int^2+3\int^3+.....]$$

$$= (1-\int)\int(1-\int)^{-2}$$

$$= \frac{\int}{1-\int}$$

$$= \frac{\lambda}{\mu-\lambda}$$

(iii) **To find the expected queue length (average number of customers in the queue or average length of waiting line)**

$$E(m) = \sum_{n=1}^{\alpha} (n-1)\,P_n\,, \qquad m = n-1$$

Since there are (n–1) units in the queue excluding one in service.

$$\therefore E(m) = \sum_{n=1}^{\alpha} n\,P_n - \sum_{n=1}^{\alpha} P_n$$

$$= \sum_{n=1}^{\alpha} n\,P_n - \left[\sum_{n=1}^{\alpha} P_n - P_0\right]$$

$$= \int(1-\int)^{-1} - \left[1-(1-\int)\right]$$

$$= \frac{\int}{1-\int} - \int$$

$$= \frac{\int - P + P^2}{1-\int}$$

$$= \frac{\int^2}{1-\int}$$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)}$$

(iv) **To find the average length of non-empty queue**

$$E(m \mid m > 0) = \frac{E(m)}{P_n(\text{an arrival has to wait, } m > 0)}$$

Now, $E(m) = \frac{\lambda^2}{1-\lambda}$

$P(m>0)=P(n-1>0)=P(n>1)$

$= P_2+P_3+P_4+ .....$

$= (\lambda^2+\lambda^3+\lambda^4+.....)P_0$

$= \lambda^2(1-\lambda)^{-1}P_0$

$= \lambda^2(1-\lambda)^{-1}(1-P)$

$= \lambda^2$

$\therefore$

$$= \frac{\mu}{\mu-\lambda}$$

(v) **To find the varience of queue length**

$$\int_0^{\alpha} \text{Var}(m)\,d\lambda = \sum_{n=0}^{\infty} n^2 P_n = \left[\frac{2^2\lambda^2}{2} + \frac{3^2\lambda^3}{3} + ...\right]_0^{\lambda} = (1-\lambda)^{-1} = \frac{1}{1-\frac{\lambda}{\mu}}$$

$$= \sum_{n=0}^{\alpha} n^2 P_n - [E(n)]^2$$

$$= \sum_{n=0}^{\alpha} n^2 P_n - \left(\frac{\lambda}{1-\lambda}\right)^2$$

Now, $\sum_{n=0}^{\alpha} n^2 P_n = \sum_{n=0}^{\alpha} n^2(1-\lambda)\lambda^2$

$= (1-\lambda)\left[1^2\lambda+2^2\lambda^2+3^2\lambda^3+.....\right]$

$= \lambda(1-\lambda)\left[1+2^2\lambda+3^2\lambda^3+.....\right]$

Let, $S=1+2^2\lambda+3^2\lambda^2+.....$

Integrating both sides with respect to λ from 0 to α, we get

$= \lambda\left[1+2\lambda+3\lambda^2+.....\right]$

$= \int (1-\int)^{-2}$

Now, differentiating both sides w.e. to $\int$ we get,

$$S = \frac{1}{(1-\int)^2} + \frac{2\int}{(1-\int)^3}$$

$$= \frac{1-\int+2\int}{(1-\int)^3}$$

$$= \frac{1-\int}{(1-\int)^3}$$

$\therefore$ v(n) =

$$= \frac{\int(1+\int)-\int^2}{(1-\int)^2}$$

$$= \frac{\int}{(1-\int)^2}$$

Waiting Times Distribution For Model 1.

Let us represent the time spent in the queue by w and the cumulative probability distribution of w be denoted by $\psi_w(t)$.

$$\sum_{n=1}^{\infty} P_n \int_0^t \frac{e^{-\mu t}(\mu t)^{n-1}}{(n-1)!} \mu \psi_w(t)dt + \psi_w(0)$$

Now, $\psi_w(t)$ =P[w=0]

= P_n (no customer in the system upon arrival]

= P_0

= $1-\int$

Now, we need to find $\psi_w(t)$ for t >0.

Let there are n customers in the system upon arrival.

Now, for a customer to go into service at a time between 0 and t, all the n customers present in the system it have been serviced by time t.

$\psi_w(t)$ =P[(n–1) customers are served at time t] λ P[one customer being served in time dt]

=

Now, since the queue length can vary between 1 and α, the p.d.f. of the waiting time is given by,

= P(w $\leq$ t), t>0

=

$$= \sum_{n=1}^{\alpha}(1-\int)\int^{n}\int_0^t \frac{e^{-\mu t}(\mu t)^{n-1}}{(n-1)!}\mu dt + (1-\int)$$

$$= (1-\int)\int\int_0^t e^{-\mu t}\sum_{n=1}^{\alpha}\frac{(\mu\int t)^{n-1}}{(n-1)!}\mu dt + (1-\int)$$

$$= \int(1-\int)\mu\int_0^t e^{-\mu(1-\int)t}dt + (1-\int)$$

$$= \int(1-\int)\mu\left[\frac{e^{-\mu(1-\int)t}}{-\mu(1-\int)}\right]_0^t + (1-\int)$$

$$= \int\left[1-e^{-\mu(1-\int)t}\right] + (1-\int)$$

$$= 1-\int e^{-\mu(1-\int)t}$$

Thus waiting time distribution is given by,

$\psi_w(t) = 1-\int$, t=0

$= 1-\int\ e^{-\mu(1-\int)t}$, t >0

Characteristics of Waiting Time Distribution

(i) Average waiting time of a customer in a queue is given by,

$E(w) = \int_0^{\alpha} t\ d\psi_{\omega}(t)$ $\int$

$$= \int_0^{\alpha} t\int\mu(1-\int)e^{-\mu(1-\int)t}dt$$

$$= \int\int_0^{\alpha}\frac{xe^{-x}}{\mu(1-\int)}dx$$ Putting, t= $\mu(1-\ )t$

$$dt = \frac{dx}{\mu(1-\int)}$$

$$= \frac{\int}{\mu(1-\int)}1(2)$$

$$= \frac{\int}{\mu(1-\int)}$$

(ii) Average waiting time of a customer who has to wait is given by,

$$E(w \mid w>0) = \frac{E(w)}{P(w>0)} = \frac{\lambda/\mu(\mu-\lambda)}{\mu/\lambda}$$

$$= \frac{1}{\mu-\lambda}$$

Illustrated Examples.

(1) In a railway mashalling yard, good trains arrive at a rate of 30 trains per day. Assuming that the interarrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes, calculate the followings :

(i) the average number of trains in the queue.

(ii) the probability that the queue size exceeds 10.

If the input of trains increases to an average 32 per day, what will happen to (i) and (ii)

Sol^{n} : Here λ = 30 per days

= trains/minute

$= \frac{1}{48}$ trains/minute

μ = trains/minute

$\therefore \quad = \frac{\lambda}{\mu} =$

(i) $E(\lambda) = \frac{\int}{1-\int} = \frac{0.75}{1-0.75} = 3$ trains

$P_r\{\text{queuesize} > 10\} = \int^{10} = (0.75)^{10} = 0.06$

when the input increases to 32 trains per day, $\frac{\int}{1-\int} = \frac{0.8}{1-0.8}$

$\lambda = \frac{1}{45}, \quad \mu = \frac{1}{36}$

$\therefore \quad = 0.8$

$\therefore$ (i) = 4 trains

(ii) P (queue size > 10)= $\int^{10}$ =$(0.8)^{10}$=0.2 (approx)

2. Arrival at a telephone booth are considered to be Poisson, with an average time of 10 minutes between one arrival and the next. The length of the phone call assumed to be distributed exponentially with mean 3 minutes. Find the followings :

(i) what is the probability that a person arriving in the booth will have to wait?

(ii) what is the average length of the queue that forms from time to time?

(iii) find the average number of units in the system.

Sol^{n} : Here, $\lambda = 1/10$

$\mu - 1/3$

(i) P_r (an arrival with have to wait) = 1–P_0

$$= 1-\left(1-\frac{\lambda}{\mu}\right)$$

$$= \frac{\lambda}{\mu}$$

$$=$$

(ii) Average length of the queue $= E(m \mid m>0)$

$$= \frac{\mu}{\mu-\lambda} = \frac{1/3}{1/3 - 1/10} = \frac{10}{7}$$

$$= 1.43 \text{ customers}$$

(iii) Average number of units in the system is,

$$E(n) = \frac{\lambda}{\mu-\lambda} = \frac{1/10}{(1/3)-(1/10)} = \frac{3}{7} = 0.43$$

3. A T.V. repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in, and if the arrival of the sets is approximately Poisson with an average rate of 10 per 8 hour day, what is the repairman's expected idle time each day? How many sets are ahead of the average set just brought?

Sol^{n} : Here, λ = 10 per 8 hour $\frac{3}{80} \times 8 = \frac{10}{60}$

= sets per minute

μ = sets per minute.

Probability that there are no units in the system is

$$P_0 = 1-\frac{\lambda}{\mu} = 1-\frac{30}{48} = 1-\frac{5}{8} = \frac{3}{8}$$

$\therefore$ Repairmen's expected idle time in 8 hour day is,

3 hours

Average number of jobs $= \frac{\lambda}{\mu-\lambda} = \frac{2}{3}$ jobs

4. An airtimes organization has one reservation clerk on duty in it's local branch at any given time. The clerk handles information regarding passanger reservation and flight timings. Assume that the number of customers arriving in a given period is Poisson distributed with an arrival rate of 8 per hour and that the reservation clerk can serve a customer in six minutes on an average, with an exponentially distributed service time.

(i) What is the probability that the system is busy?
(ii) What is the average time a customer spends in the system?
(iii) What is the average length of the queue and what is the number of customers in the system?

Soln : Here, = 8 customers/hour

= customer/minute

$= \frac{60}{6} = 10$ customer/hour

$\therefore \quad = \frac{\lambda}{\mu} = \frac{4}{5}$

(i) The probability that the system is busy is given by,

$= \frac{\lambda}{\mu}$

$= 0.8$

(ii) The average time a customer spends in the system is,

hours

(iii) Average length of the queue

$E(m/m>0) = \frac{\mu}{\mu-\lambda}$

=

= 5 customer.

Average number of customers in the system is,

$\frac{\lambda}{\mu-\lambda} = \frac{8}{10-8} = 4$ customers

Check your progress

1.Derive the differential difference equations for the queuing model (M|M|I) : (|FCFS). How would you proceed to solve the model?
2. Explain the M|M|I queuing model in transient state. Derive the steady state solutions.
3. In (M|M|I) : (|FCFS) model obtain p.d.f. of waiting time.
4. Obtain the steady state solution of (M|M|I):(|FCFS) queuing model.
5. In a telephone booth the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. If there is just one phone :

(i) What is the expected number of callers in the booth at any time?
(ii) For what proportion of time is the booth expected to be idle?

6. In a bank there is only one window. A solitary employee performs all the service required and the window remains continuously open from 7A.M. to 1P.M. It has been discovered that the average number of customers is 54 during a day and that the average service time is five minutes per person. Calculate.
(i) average number of clients in the system (including the one being served).
(ii) the average number of clients in the waiting line (excluding the one being served).
(iii) the average waiting time.

7. At a public telephone booth the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. If there is just one phone,
(i) What is the expected number of callers in the booth at any time?
(ii) For what proportion of time, the booth expected to be idle?

BLOCK IV : UNIT-I

Paper Name: Operation Research and Computer in Business-M. Com. 2nd sem

Part-2 Computer in Business

Block 4: System Analysis and design

Unit Structure:

1.1 Introduction

In business, System Analysis and Design refers to the process of examining a business situation with the intent of improving it through better procedures and methods. System analysis is a process of employing various approaches to the study and solution of problemsusing computer-based systems. The components of a system include the system elements, processes and technology. An organization is considered a complex system that consists of interrelated and interlocking subsystems. The systems approach is a way of thinking about the analysis and design of computer-based solutions. Before introducing a computer-based system, it is very much required to know about the procedure of functioning of an organization. As for example, in an airport scenario, knowledge of traffic flow, the location of the airport, etc. are important factors in deciding on improvements of functioning

of the airport. Working with managers and employees in the organization, systems analysts recommend which alternative to adopt, based on such concerns as the suitability of the solution to the particular organization and setting, as well as the employee support the solution is likely to have. Sometimes the time required to develop one alternative, compared with others, is the most critical issue. Costs and benefits are also important determinants. In the end, management, which will pay for and use the result, actually decides which alternative to accept. Once this decision is made, a plan is developed to implement the recommendation. The plan includes all systems design features, such as new data capture needs, file specifications, operating procedures, equipment and personnel needs. This unit will describe the system concepts, elements of a system, various types of a system which are relevant to system analysis.

1.2 Objectives:

At the end of this unit, you will be able to learn

i. The characteristics of a system and the importance of the systems concepts.
ii. How the various elements of a system work together.
iii. Various types of system.
iv. The concept of man-made information system.

1.3 Concept of System:

The term System is derived from the Greek word systema which is meant for an organized relationship among various functional units. A system is an orderly grouping of interdependent components linked together according to a plan to achieve a specific objective. The word component may be physical parts, managerial steps, or a subsystem. The study of system concepts has three basic objectives:

1. A system must be designed to achieve a predetermined objective.
2. Interrelationships and interdependence must exist among the components.
3. The objectives of the organization must be prioritized than the objectives of its subsystems.

1.4 Characteristics of a system:

A system offers some characteristics which include the following:

- **Organization:**

It implies structure and order. It is the arrangement of various organizational components that help to achieve objectives. In the structure of a business system, the hierarchical relationships starting from Chairman cum Managing Director leading down to the assisting staff represents the organizational structure.

- **Interaction:**

It refers to the style in which each component does interact with other components of the system. For instance, in a big organization, purchasing section has to interact with production, advertising with sales, and payroll with personnel.

- **Interdependence:**

This feature states that parts of the organization or computer system depend on one another. These parts are coordinated and linked together according to a plan. For proper functioning, one subsystem must depend on the output of another subsystem.

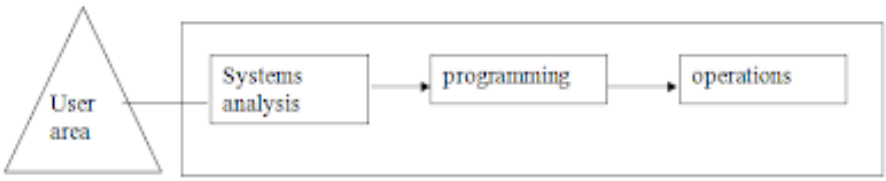


Figure 1.1 Interdependence of parts of an organization

No subsystem can function in isolation. A decision to computerise an application is originated by the user, followed by analysis and design by the analyst, programmed by the programmer and run by the operator as presented in figure 1.1.

- **Integration:**

  It is concerned with how a system is tied together and thereby it offers to the holism of systems. It means that various parts of a system work together within the system though each part individually works.

- **Central Objective:**

A system should have a central objective. Objectives may be real or stated. Although a stated objective may be the real objective, it is common for an organization to

mention one objective and operate to achieve another. The important point is that users must know the central objective of a computer application early in the analysis for a successful design and conversion.

1.5 Elements of a system

Usually, system analyst works in a dynamic environment which may be a business firm or a computer system. In order to restructure a system,three basic elements input, processor(s) and output must exist in a system. Moreover, system may include other elements such as control, feedback, boundaries, environment and interfaces.

- **Input**: Input is what data the system receives to produce a certain output. Inputs may be material, human resources and information that are fetched into the system for processing.
- **Output**: What goes out from the system after being processed is known as Output. Output must be in line with the requirements of the intended user.
- **Processor:** The processor is the main element which is involved to transform input into output. Processors may change the input totally or partially, depending on the specifications of the output.
- **Control:** As a decision-making subsystem, it monitors and controls the input, processing and the output of the system in order to get the desired results. For example, management is considered decision-making body in an organization that controls the inflow, handling and outflow of various activities; in a computer system, operating system provides the platform for performing other activities.
- **Feedback:** The output is checked with the desired standards of the output set and the necessary steps are taken for achieving the output as per the standards. This process is called as Feedback. It helps to achieve a much better control in the system.Feedback may be in the form of positive or negative, routine or informational. In system analysis, feedback is important in different ways which may result in enhancements in order to meet the user's requirements.
- **Boundaries:** The boundaries are nothing but the limitation of the system that identify its components, processes and interrelationships when it has interfaces with other systems. Setting up boundaries helps for better concentration of the activities carried out in the system.For example, a teller system in a banking organization is

restricted to deposits, withdrawals and related activities of customers' checking and different accounts.

- **Environment:** The environment refers to the suprasystem within which an organization works. The things outside the boundary of the system are known as environment. Change in the environment affects the working of the system.
- **Interfaces:** The interconnections and the interactions between the sub-systems is known as the Interfaces. They may be inputs and outputs of the systems.

Check Your Progress

1. A system is an orderly grouping of _____ components linked together according to a plan to achieve a specific objective. (Fill up the blanks)

2. The processor is the element of a system to transform the input into the desired output. (State True or False)

3. Which feature states that parts of the organization or computer system depend on one another?

1.6 Types of System

Systems can be classified in the following ways:

- ✓ **Physical and Abstract**
- ✓ **Open and Closed**
- ✓ **Permanent and Temporary System**
- ✓ **Deterministic and Probabilistic**
- ✓ **Natural and Man-Made System**
- ✓ **Man-made Information System**

1.6.1 Physical and Abstract systems:

Physical systemsare termed as tangible entities and they may be static or dynamic in nature. The Office room, Computer table and Chair are some static physical objects whereas a programmed computer can be considered a dynamic entity. Abstract system

is conceptual or non-physical entity. The abstract is conceptualization of physical situations. For example, formulas of relationships among sets of variables belong to non-physical entities.

1.6.2 Open and Closed systems:

Based ontheirdegree of interdependence**,** systems may be open or closed.An open system continually interacts with its environment. Inputs from the outside are receivedby the system and forwards output to the outside world. A closed system is isolated from environment influences.

Open system has five important characteristics which are input from outside, entropy, differentiation, Equifinality and Process, output and cycles.

Other systems can be defined as:

- ✓ **Permanent and Temporary System:** A permanent system is a system enduring for a time span that is long relative to the operation of human. Temporary system is one having a short time span.
- ✓ **Natural and Man-Made System:** System which is made by man is called man made system. Systems which are in the environment made by nature are called natural system.
- ✓ **Deterministic and Probabilistic:** A Deterministic system is one in which the occurrence of all events is perfectly predictable. If we get the description of the system state at a particular time, the next state can be easily predicted. Probabilistic system is one in which the occurrence of events cannot be perfectly predicted.

1.7 **System models**

A model represents a real or a planned system. A system analyst can visualize relationships in the system under study by using a model. Various system models are proposed to demonstrate the benefits of how to abstract a complex system to a model form. These major models include schematic model, flow system model, static system model and dynamic model.

Flow System Models

This type of model shows the flow of the material, energy and information that hold the system together.There is an orderly flow of logic in such models. A widely known example is PERT (program evaluation and review technique).It is used to abstract a real-world system in model form, manipulate specific values to determine the critical path, interpret the relationships and relay them back as a control. The probability of completion with in a time period is considered in connection with time, resources and performance specifications shown in the figure 1.2.

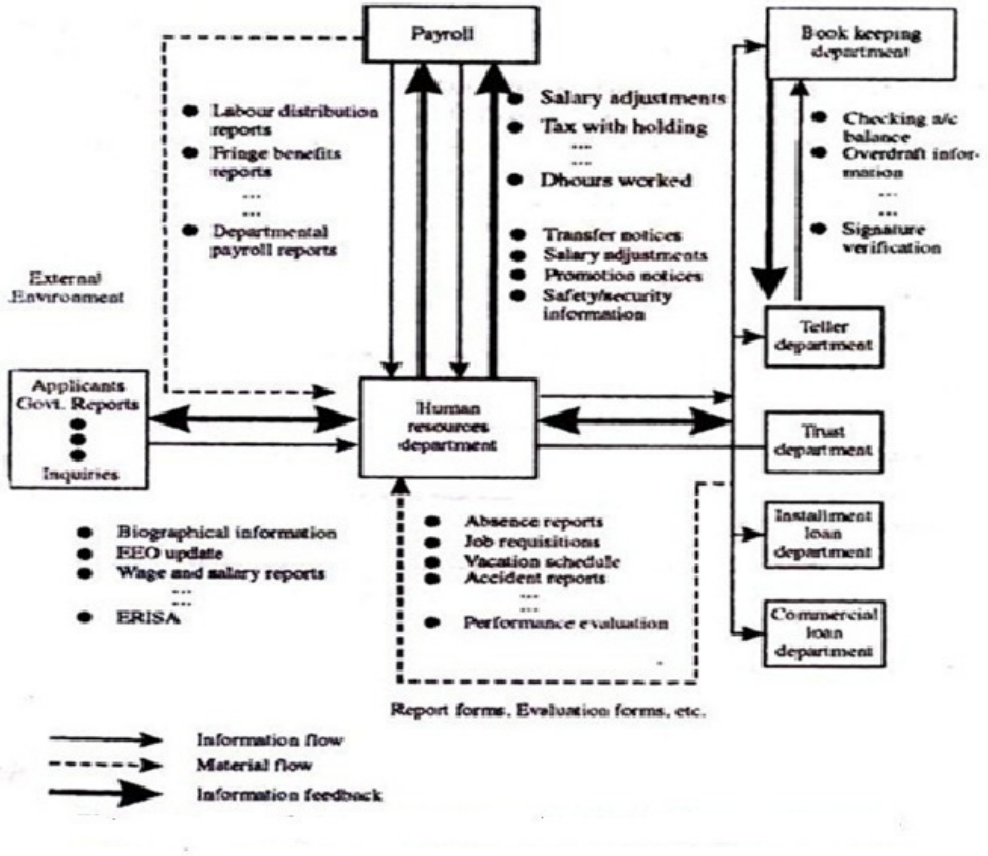


Figure 1.2 The Information flow

Schematic Models

This type of model is a two-dimensional chart depicting system elements and their linkage. Figure 1.2 shows the major elements of a personnel information system together with material and information flow.

Dynamic System Models

Business organisations are dynamic systems. A dynamic model approximates the type of organisations or applications that analysts deal with. It depicts ongoing, constantly changing system.

Dynamic system model goes through the following steps:

1. It inputs which enter the system.
2. The program(s) required for processing.
3. The output(s) that result from processing.
4. The processor through which transformation takes place.

Static System Models

This type of model shows one pair of relationships such as activity time or cost quantity. The Gantt chart, for example, gives a static picture of an activity time relationship. As presented in the figure 1.3, planned activities are plotted in relation to time in Gantt Chart. The data column has light lines that indicate the amount of time it takes to complete a given activity The bold line represents the cumulative line schedule for each activity.

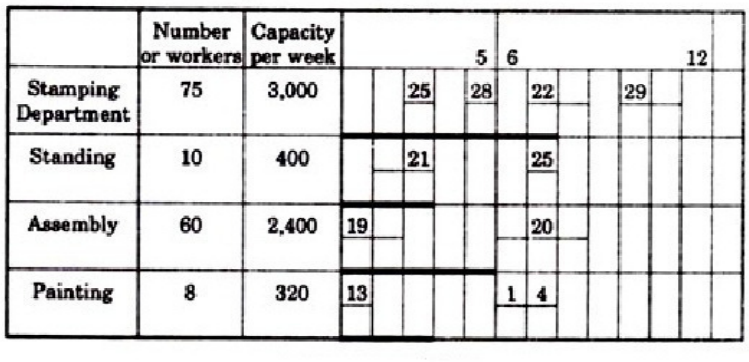


Figure 1.3 Gantt Chart

The stamping department, for example is schedule to start working on order number 25 Wednesday morning and complete the job by the same evening. One day is also scheduled for order number 28, two days for order number 22, and two days (May 10-11) for order No. 29.

The total of 6 days is represented by the bold line opposite the stamping department. The broken line indicates that the department is two days behind schedule.

1.8 Man-made Information System:

It is generally believed that the information reduces uncertainty about a state or event. An information system is the basis for interaction between the user and the analyst. It fixes the nature of relationship among decision makers. From this perspective, an information system can be well-defined as a combination of devices, procedures and operating structure designed around user-base criteria to produce information and delivering it to the user for planning control and performance.

There are three major classifications for information system.
1. Formal information system
2. Informal information system
3. Computer based information system

Formal information system

A formal information system is based on the organization represented by the organization chart. The chart is a map of positions and their authority relationships, indicated by boxes and connected by straight lines. It is concerned with the pattern of authority, communication and workflow. Information is formally disseminated in instructions, memos or reports from top management to the intended user in the organization.

There are three categories of formal information system
1.strategic information system
2.managerial information system
3.operational information system

Strategic information system relates to long range planning policies. These are the direct interest of the upper management. Polices are the generalization that specify what an organization is going to do, for example, financial investment in a company, human resources etc.

Managerial information system relates to short and intermediate range planning policies.This is done by the direct interest of middle management and department heads

for implementation and control. The duration will take several months to year.

Operational information system relates to short term polices. This deals with day-to-day activities like daily employee absence sheets, purchase orders, current stock available for sale.

INFORMAL INFORMATION SYSTEM

It is an employee-based system designed to meet personal and vocational needs and to help and solve work related problems. It also funnels information upward through indirect channels. It is a useful system because it works within the framework of the business and its stated policies.

1.8.1 *COMPUTER BASED INFORMATION SYSTEM (CBIS)*

Computer-Based Information System is essentially an information system using computer technology to carry out some or all of its planned tasks. A CBIS is an organised integration of hardware and software technologies and human elements designed to produce timely, integrated, accurate and useful information for decision making purposes. It is the combination of people, hardware, procedure, software and data which make up a computerized system that can interpret data into useful information, developed by the people and for the people. A CBIS is an information system in which the computer plays a major role and the data is mostly collected, stored, and processed in digital format using computerized processes. Similarly, the information is delivered in digital format via computerized mechanisms. There are five components in CBIS which is hardware, software, data, people and procedures. The components work together to form a computer-based information system. People seems to be the most important component in CBIS. The basic components of computer-based information system are:

- **Hardware:** The term hardware refers to machinery. This category includes the computer itself, which is often referred to as the central processing unit (CPU), and all of its support equipment's. Among the support equipment's are input and output devices, storage devices and communications devices which are shown in figure 1.4.

Figure 1.4. Various hardware devices

- **Software:** The term software refers to computer programs and the manuals (if any) that support them. Computer programs are machine-readable instructions that direct the circuitry within the hardware parts of the CBIS to function in ways that produce useful information from data. Programs are generally stored on some input / output medium-often a disk or tape.
- **Data:** Data are facts that are used by program to produce useful information. Like programs, data are generally stored in machine-readable from on disk or tape until the computer needs them.
- **Procedures:** Procedures are the policies that govern the operation of a computer system. "Procedures are to people what software is to hardware" is a common analogy that is used to illustrate the role of procedures in a CBIS.
- **People:** Every CBIS needs people if it is to be useful. Often the most over-looked element of the CBIS is the people: probably the components that most influence the success or failure of information system.

Types of Computer-Based Information Systems:

Transaction Processing Systems

The most fundamental computer-based system in an organisation pertains to the processing of business transactions. A transaction processing system can be defined as a computer-based

system that captures, classifies, stores, maintains, updates and retrieves transaction data for record keeping and for input to other types of CBIS. Transaction Processing Systems are aimed at improving the routine business activities on which all organizations depend. A transaction is any event or activity that affects the whole organisation. Placing orders, billing customers, hiring of employees and depositing cheques are some of the common transactions. The types of transactions that occur vary from organisation to organisation.

But this is true that all organisations process transactions as a major part of their daily business activities. The most successful organisations perform this work of transaction processing in a very systematic way. Transaction processing systems provide speed and accuracy and can be programmed to follow routines without any variance.

Management Information System

Data processing by computers has been extremely effective because of several reasons. The main reason being that huge amount of data relating to accounts and other transactions can be processed very quickly. Earlier most of the computer applications were concerned with record keeping and the automation of routine clerical processes. However, in recent years, increasing attention has been focused on computer applications providing information for policy making, management planning and control purposes. Management Information System are more concerned with management function. MIS can be described as information system that can provide all levels of management with information essential to the running of smooth business. This information must be as relevant, timely, accurate, complete and concise and economically feasible

Decision Support Systems

It is an information system that offers that kind of information which may not be predictable, the kind that business professionals may require only once. These systems do not produce regularly scheduled management reports. Instead, they are designed to respond to a wide range of requests. It is true that all the decisions in an organisation are not of a recurring nature. Decision support systems assist managers who must make decisions that are not highly structured, often called unstructured or semi-structured decisions. A decision is considered unstructured if there are no clear procedures for making the decision and if not all the factors to be considered in the decision can be readily identified in advance. Judgement of

the manager plays a vital role in decision making where the problem is not structured. The decision support system supports, but does not replace, judgement of manager.

Office Automation Systems

Office automation systems are among the newest and most rapidly expanding computer-based information systems. They are being developed with the hopes and expectations that they will increase the efficiency and productivity of office workers-typists, secretaries, administrative assistants, staff professionals, managers and the like. Many organisations have taken the First step toward automating their offices. Often this step involves the use of word processing equipment to facilitate the typing, storing, revising and printing of textual materials. Another development is a computer- based communications system such as electronic mail which allows people to communicate in an electronic mode through computer terminals. An office automation system can be described as a multi-function, integrated computer-based system that allows many office activities to be performed in an electronic mode.

Categories of different information systems with their characteristics have been listed briefly in the following table 1.

Category of Information System	**Characteristics**
Transaction Processing System	Substitutes computer-based processing for manual processes. Includes record-keeping applications.
Management Information System	Provides input to be used in the managerial decision process. Deals with supporting well-structured decision situations. Typical information requirements can be anticipated

Decision Support System	Provides information to managers who make judgements about particular situations. Supports decision makes in situations that are not well structured.
Office Automation System	It is a multi-function, integrated computer-based system, that allows many office activities to be performed in an electronic mode.

Table 1. Different categories and characteristics of information systems.

1.9 Summary

- A system is an orderly grouping of interdependent components linked together according to a plan to acquire a specific objective. The main characteristics of system are organization, interaction, interdependence, integration and a central objective.
- System analysis and design refers to the application of the system approach to solve a problem using computers. To reconstruct a system, the analyst must consider its elements like inputs and outputs, processors, control, feedback and environment.
- System falls under three classes:
    i. Physical or Abstract
    ii. Open or closed
    iii. Man-made information system
- A system model may be schematic (two-dimensional), flow system to abstract the real world, a static or a dynamic system model.
- An information system is an open system that allows inputs and facilitates interaction with the user.
- There are three levels of information in organization:
    i. Strategic information relates to long-range planning policies and upper management. It is extracted with decision support system

ii. Managerial information helps middle management and department heads in policy implementation and control.

iii. Operational information is daily information required to operate the business.

- Computer-Based Information System is essentially an information system using computer technology to carry out some or all of its planned tasks. A CBIS is an organised integration of hardware and software technologies and human elements designed to produce timely, integrated, accurate and useful information for decision making purposes.
- Management information system (MIS) provides input to be used in the managerial decision process.
- The key element of MIS is the database which refers to a nonredundant collection of interrelated data items which are processed through application programs.

1.10 Suggested readings

Elias M. Awad, "Systems Analysis and Design", second edition, Galgotia publication

1.11 Model Questions:

1. Define system. Give examples.
2. What are the elements of a system? Do you have any idea about a viable system without feedback? Explain it.
3. Distinguish between:
    a. Physical and abstract systems.
    b. Schematic and Static system models
    c. Interaction and interdependence.
4. Explain the features of a system.
5. Elaborate the different types of systems.
6. What is man-made information system?
7. A system leads to a lot of planning and less of implementation. Do you agree? Justify your answer.
8. What type of information is relevant to decision making in business? Discuss it.
9. Discuss the concept of Computer-based information system.
10. Why is database important in MIS? Explain.

BLOCK IV : UNIT-II

Information System Development

Unit Structure:

2.1 INTRODUCTION

Information Systems Development is the process that is used to develop and maintain computer based information systems. Systems Development could include the creation of database systems, internal development of customized systems, or even the acquisition of

software developed by third party. The design and analysis of information systems are done from an organizational point of view. Systems development is an organizational improvement process**.** For the purpose of organizational benefits, systems are built and rebuilt .Benefits result while values get added during the process of creation and production of the services and products of the organization. Organizations find it helpful to use systems development methodology which is a standard sets of steps, to develop and support their information systems (IS). Similar to many processes, the Information Systems' development often follows a life cycle called Systems Development Life Cycle (SDLC).

For example, a product conforms to a definite life cycle when it is created, tested and launched in the market. Along the way, its sale increases and reaches the peak point and then declines. Then, a new product altogether or a version of the existing product is launched to replace it in the market. SDLC is a common process for systems development in different organizations, which includes several phases to mark the progress of the system analysis and design tasks. .

2.2 OBJECTIVES

Completing this unit will enable you to:
• define what System development is;
• know about the entire Process of System Development
• explain the information systems development Life Cycle
• know about the Systems development methodologies

2.3 Systems Development Process

The **system development process**, often referred to as the SDLC or system development life cycle is a set of methodical activities, processes, or phases that are used to develop a system and implement it. With the rapid development and continuous evolution of technologies till date, many different development methodologies have been formulated and tested. Even though each one of these methodologies has been developed keeping in mind a specific design, but it has been discovered that most of them share common tasks and reflect a common goal. A system or more specifically a software system attempts to solve a problem related to an organization. Certain common steps are followed in the process of developing most of the systems. These steps also describe the general details taken to achieve this. Each step is designed in order to develop a system that is characterized by accuracy and quality and one that reflects the requirements of the client. The steps are described as follows:

2.3.1. Information Gathering

In this step, the problems are studied at first. Then the objectives are chalked out and the resources required to fulfill these objectives are outlined. All the participating stakeholders such as the clients, developers, end users, and consultants come together and engage in brainstorming. If the software is brand new then more information and data gathering will be required .On the other hands if the system is not brand new then more focus will be on improvements than on and data gathering.

2.3.2. Analysis

At this stage, the requirements of the end-user are clearly formulated. In order to analyze the impacts of the proposed system, feasibility studies are carried out. Information regarding competitive products is also analyzed. And once the viability of the project is assured the project is divided into workable segments.

2.3.3. Design

With well defined workable segments, the proposed system is designed. Customer requirements form the different elements of the system being designed. Functionality details of each element, the techniques, and the logic of each of the processes is prepared.

2.3.4. Implementation

In this phase the actual system is constructed. The logical part of all the segments comprising the system is developed and building of any hardware required is accomplished. The codes are written using programming language decided to use. The individual programs are also tested as they are developed

2.3.5. Deployment

The development of the system is completed and is ready to be deployed along with any accompanying hardware required. The system developed which guarantees to fulfill the requirements of the customer is installed at the customer's end .The complexity of the system often decides the time for it to actually go live. Training is organized for the end-user to ensure that they can properly use the system.

2.3.6. System Testing

The different parts of the system are assembled together, making a complete system. Different inputs are collected from the user, and fed into the system. Real or end-users are often employed to carry out the system testing. The testing phase is aimed to ensure that all the customer's requirements are met .This phase also ensures that overall customer satisfaction is achieved. No knowledge of hardware or software, or technical expertise is needed. In order to conduct in-depth and robust testing of the system few specialized

Stop to consider

Systems development process is the methodical activity comprising of defining, designing, testing, and implementing a new software system

2.4 SYSTEMS DEVELOPMENT LIFE CYCLE (SDLC)

Most of the organizations find it helpful to follow a set of steps, called the systems development methodology, for developing and supporting their information system. Similar to most processes, the development of an information system often follows a life cycle. SDLC (or system development life cycle) is a common methodology adopted for developing

a system in many organizations. It specifies the various phases that reflect the progress of the system analysis and design effort.

Although, at first glance, it seems that any life cycle is a serially ordered set of phases but in reality it is not. The particular steps and their subsequent sequence are intended to get adapted as required for a specific project which is consistent with management approach. In any system development life cycle phase, the project can go back to an earlier step, as and when need arises. For instance, if a product performs poorly after being introduced into the market, it may temporarily be withdrawn from the market and re-introduced later with improved features. Moreover, in SDLC it is also possible to parallelly carry out two different activities that belong to two different phase of the life cycle. At times, life cycle become iterative; that is, some phases are repeated until an acceptable and satisfactory system is developed. Such kind of iterative approach forms a special characteristic of RAD or rapid application development tools, such as prototyping. Furthermore, some people also consider software development life cycle to be spiral as we have to repeatedly cycle through some of the phases at different stages of development of the system. The system development life cycle can also be considered as a circular process in which the end of one system's useful life leads to the start of another project ,that will either develop a new version of the system by incorporating new features or completely replace the existing system. Nevertheless, the system development life cycle implemented in any organization is an ordered set of activities that is planned and conducted for the development of each project. A system analyst 's skills are required for applying throughout the life cycle.

Each of the custom software producer will have their own system development methodology or specific life cycle. Even if the particular methodology does not appear to be a cycle, it can be discovered that most of the steps in SDLC are performed, the techniques and tools therein are used. With a view to make this unit more generic, we follow here a relatively general life cycle model, as described in figure 2.1.

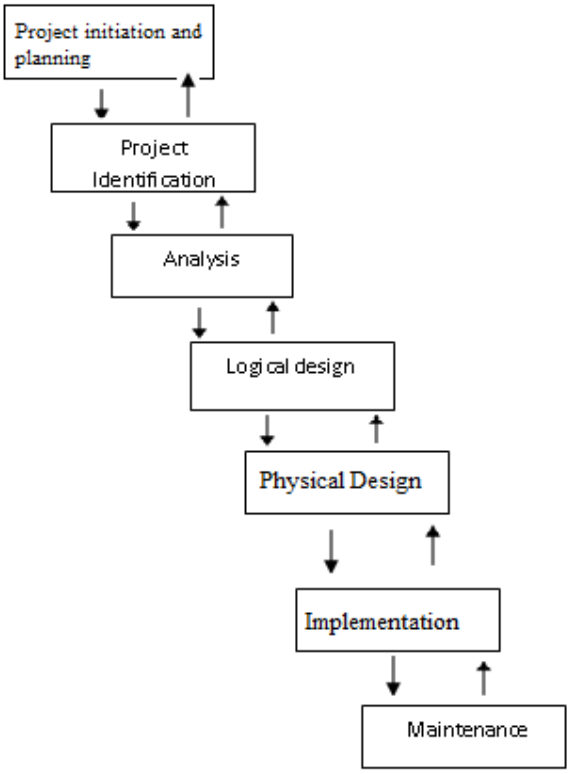


Fig 2.1 Phases of System Development Life cycle

Stop to consider
Every organization follows a specific methodology for developing an information system, with most of the steps bearing resemblance to phases in SDLC

The model in the figure bears a resemblance to a staircase with the arrows connecting each step to the step before and after it. This representation of SDLC (system development life cycle) is sometimes called the "waterfall model". We shall use this system development life cycle not only as one example methodology but also to represent it as a means to arrange the steps of systems analysis and design. At the end of each phase, specific outputs and deliverables are generated that provide important information for starting the next phases. System development project reaches a milestone at the end of each phase, and, because deliverables are produced, members not from the project team often review them.

Steps in SDLC

SDLC (system development life cycle) comprise of the following steps:

1. Project Identification and Selection
2. Project Initiation and Planning

3. Analysis
4. Logical Design
5. Physical Design
6. Implementation
7. Testing.

2.4.1 **Project Identification and Selection**

Project identification and selection is the first phase in the SDLC. This phase starts with identification of the user's need for a new or improved system. In some organizations, this identification forms a part of a systems planning process. Information pertaining to the requirements of the organization is inspected. In order to meet these requirements , projects are thereby proactively identified. The information system requirements of the organization may be the result of requests to solve the problem faced in the current system's procedures, or the organisation's desire to perform additional tasks, or from the realization that in order to capitalize on an existing opportunity ,information technology could be used. These needs are then prioritised and translated into a plan for the Information System department .This also includes a schedule for the new major system's development. In case of smaller organizations, ascertainment of which systems to develop may get affected by request submitted by the user to develop a new or an enhanced systems as well as from a formal information planning process. In either of the cases, during this phase, an organization assesses whether or not to devote resources to the enhancement or development of each information system under consideration. The result of the project identification and selection step is the determination of a systems development projects that needs to be undertaken by the organization at least in terms of an initial study.

Stop to consider
The Software Development Life Cycle starts with the Project Identification and Selection step.

2.4.2 Project Initiation and Planning

The second phase of SDLC is the project initiation and planning. The problems that are identified in the previous phase needs to be investigated and the organization must come to a decision whether to implement the information system or not. An important step at this stage is determining the proposed system's scope. The project leader and the initial team members of the system analysts design a specific plan for the proposed project. The team will have to follow this plan in the remaining SDLC steps. This standard SDLC is customized in this initial project plan. The project plan also specifies the time and resources required for its execution.

A project's formal definition is based on the whether the information system department of the organization needs to develop a new system in order to solve the problem or use the available opportunity .It also depends on calculating whether the costs incurred in developing the system outweighs the benefits that it could provide. Usually the project leader and some of the members from the team give the final presentation as this project phases proceeds to

the management or to a special management committee entrusted with the responsibility of deciding which project should be undertaken by the organization.

Stop to consider
An important step in Project initialization and planning is determining the scope of the project

2.4.3 Analysis

Analysis is the third phase in the SDLC. An important question is- what must be done in order to solve the problem? One facet of analysis is that it defines the boundaries of the system and determines whether a candidate system should or should not consider other related systems. This phase consist of several sub-phases the first sub-phase is identification of requirements. In this sub-phase, the system analysts work with the users in order to determine the need and expectations of users from the candidate system. It generally involves a meticulous study of the current systems, whether manual or computerized system that might either be replaced with or enhanced as part of the project. Data are collected on the available files, and transactions handled by the present system. Some of the analysis tools generally used for the purpose are data flow diagrams, questionnaires, on – site observations and interviews. Interviews are one of the commonly used tools in analysis. It requires special expertise and responsive to the subjects being interviewed. Biasness can be a problem both in data collection and its interpretation. Experience and common sense both play an important role for collection of the information needed to carry out the analysis. The requirements are studied in detail and are structured in accordance with their inter-relationships and redundancies are eliminated. Third, an alternative design is generated meeting the requirements. These alternatives are then, compared to decide the best one that meets the requirement in terms of cost and labor to carry out development process.

During this sub-phase, feasibility study of the candidate system is also performed. The various types of feasibilities are as follows:

2.4.3.1 Technical feasibility:

In this phase the current resources available, both hardware and software, along with required technology are analyzed/assessed to develop project.

2.4.3.2 Economic feasibility

In economic feasibility the cost that will be incurred in the proposed project and the benefits that will be derived from it is studied.

2.4.3.3 Behavioral feasibility

It analyses the user's attitude or behavior towards the proposed system.

2.4.3.4 Operational feasibility

In this feasibility study the degree of service that can be provided to requirements along with how much easy it will be to operate the product and to maintain it after its deployment is analyzed.

2.4.3.5 Legal feasibility

Legal Feasibility Study is the study to determine if the proposed project conforms to the legal and ethical requirements.

2.4.3.6 Time feasibility.

This feasibility study analyzes the deadline or timeline set for the proposed project.

If the proposed system is found not feasible to develop, it is rejected at this step. The analysis phase outputs a description of the alternative solution proposed by the analysis team.

Stop to consider
The feasibility study decides whether to carry out the proposed system or not

System Design: After analysis phase, the design of the system begins. It consists of logical and physical design of the new system. The next two phases (fourth and fifth) concentrates on the design of the new or enhanced system. During the said design, the detailed description of the recommended and alternative solution is converted into logical and physical specifications. All aspects of the system from input-output screens to reports, computer processes and databases must be designed. Usually design occurs in two phases, logical design and physical design.

2.4.4 Logical Design

Logical design is not limited to a specific systems software and hardware platform. Theoretically, a system could be implemented on any systems software and hardware. The idea is to ensure that the system functions as expected. This phase concentrates on the business aspects of the system.

2.4.5 Physical Design

In this phase, the logical design is transformed into physical or technical specifications. For example, diagrams that map the origin, flow and processing of data in a system needs to be converted into a structured systems' design that can then be decomposed into smaller and smaller units called modules so that this could easily be translated into instruction using a suitable programming language. The various parts of the system are designed to perform the physical operations such as data capture, processing, and information output. During this phase the analyst team decides the programming language to be used for coding, the database system and file structure to be used, the platform and the networking environment under which the system will run. These decisions help in finalizing the hardware and software plans. The design phase, as its final product, outputs the physical system specification which is handed over to programmers and other system builders for construction. The physical

system specifications are sent to the programmers during the first part of the implementation procedure.

Check Your Progress

1. What is the difference between Project Identification and Project Initiation?
2. What is the difference between analysis and design?
3. Why do most of the projects die after feasibility phase?

2.4.6 Implementation

During implementation phase, the system specification is translated into a working system that is repeatedly tested and put into use. Implementation process includes coding the system, testing and installing it. Programmers write programs that build up the system. The individual programs and the entire system are then tested by programmers and analysts in order to detect errors, if any, and correct them. The new system is installed on the existing or new hardware .Users are introduced to the newly developed system and trained accordingly so that the daily activities of the organization can be smoothly carried out.

Installation of the new system can be carried out in the following ways:

• Direct conversion:

In this conversion, the software is directly installed at user's site.

• Parallel conversion:

In Parallel conversion type, the old system continues to run parallelly with new systems for some time. After observing the performance of the new system for a reasonable period of time , if found satisfactory, then, the old system is replaced by the new system one.

• Phased conversion:

The system is installed one module at a time in phased conversion

Implementation activities also consist of the finalization of documentation, training programs, and user assistance. Documentation is generated throughout the lifecycle. In spite of the best efforts of analysts, programmers, and managers, however, installation process is not always a simple one. The reason behind many well-designed systems' failure is that implementation is not properly carried out by the project team.

Stop to consider

Implementation includes coding, testing and installation

2.4.7 Maintenance

The final phase of SDLC is maintenance. After installation when a system continues its operation, in an organization, users sometimes encounter problems relating to how it operates, and often think of better solutions. Also, the organization's needs with respect to the system change with the passage of time. During the maintenance phase, programmers make changes in the programs to meet the users' new requirements and modify the system accordingly to reflect the changing business conditions as these changes are essential to make the system running and be useful to the organization. Maintenance is not a separate phase. It is just a repetition of some of the phases of the lifecycle so as to study and bring about the required changes. The amount of effort and time required required for maintenance depends to a great extent on how well the previous phases of the life cycle performs. However, a time will come, when that particular information system will no longer perform as is desired, when the organization's requirements changes substantially when maintenance cost become exorbitant. Such situations are indicative of the fact that the time has come to replace the existing information system and hence completes the loop and marks the start of the life cycle over again.

Stop to consider
Maintenance is an overlay to the life cycle rather than a separate phase

Three types maintenance are there. They are:

2.4.7.1 Corrective maintenance:

In this type of maintenance, errors that crop up in the system are removed.

2.4.7.2 Adaptive maintenance:

In this type of maintenance, errors are adapts with the changing external factors. For example, if the Dearness Allowance increases from 100% to 110%, then in order to adapt to these changing scenario, changes also have to be made in the Information System.

2.4.7.3 Perfective maintenance:

In order to satisfy user's requirements and thereby make the system more perfect then perfective maintenance is carried out.

SDLC is a highly connected set of phases where the output of one phase serves as input to the subsequent phases. The systems development project needs to be carefully planned and developed during each phase of the life cycle.

Check Your Progress

1. What are the different types of maintenance?
2. Describe the process of Implementation?

2.5 System Development Methodology

System Development Methodology is the standard procedure followed in an organization to perform all the steps essential for analyzing, designing, implementing, and maintaining information systems

In the continuous effort aimed at improving the overall process of systems analysis and design, a number of approaches have been developed. To make system development more of a science and less of an art, usually referred to as engineering techniques, are also applied to the process of system development. This section will discuss prototyping, joint application design and participatory design.

2.5.1 Prototyping

Prototyping is the software development methodology focused at the use of working models that constantly refines itself based on the end user's feedback. Designing and building a small but fundamental version of the desired system is also known as prototyping

Prototyping is generally used to develop information systems with significant user interaction and also complex user interfaces.

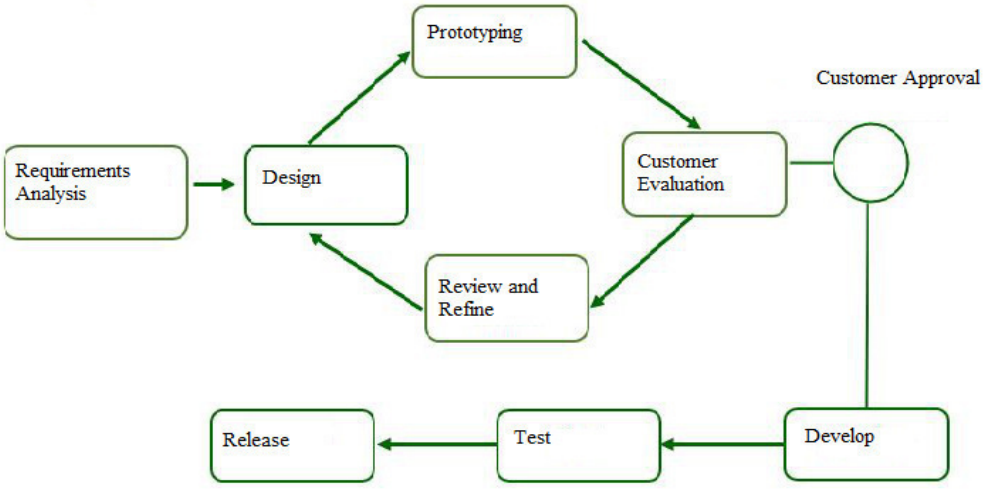


Fig 2.2 The prototype model

When using prototyping as the development methodology, the system analyst works with the end-user to identify the basic or initial needs of the system. With the initial requirements in hand the analyst then creates a prototype of the system. Once the prototype is completed, it is handed over to the user to work with it .The end –users then give their feedbacks to the analyst about what they disliked and what they liked about it. These feedbacks are then used by the analyst to upgrade the prototype and this new version is again given to the user to work with. This whole process is iterated until the end-users are fully satisfied. Prototyping technique has few advantage one being the large extent to which it involves the user in the analysis and design process which helps in understanding the user-specific requirements and the other being its ability to record the requirements in concrete form rather than verbal one.

Prototyping is a form of RAD or rapid application development. The basic principle of any rapid application development methodology is to delay the process of producing complete system design document till the end-user requirements are clear.

A prototype can be built using any development tool or computer language to simply the process of development. The prototype of a system can be built using some 4GLs (fourth generation languages) or with tools called CASE (computer aided software engineering) tools.

Stop to consider
When the user's needs are not clear, the prototype model is used as a methodology fro the information systems development

2.5.2 Joint Application Design

Joint Application Design was developed in the late 1970s by the systems development personnel working at IBM .It was a new process for collecting requirements for information system and for reviewing systems designs. To meet the business needs, it collects requirements parallelly with the development of new information systems. The fundamental idea behind Joint Application Design is to structure the requirements determination phase (a sub-phase of analysis phase) and the reviews that take place as part of the design phase. End-users, managers, and system developers sit together for series of structured meetings with a JAD session leader conducting the intensive meeting. He is the one who sticks to the agenda and maintains the structure. By bringing all the people directly affected by the Information System sit together it helps them to come to an agreement on the system requirements and design details. This also aids in better use of time and organizational resources. In addition to this, the group members also get a grasp of what the information system is supposed to do.

Generally in a JAD life cycle, there are four participants – the Executive Sponsor, the IT Representative, Scribe and the user. The Scribe is entrusted with the responsibility of documentation and also to act as the facilitator for carrying out JAD Sessions. JAD life cycle consist of the following phases: Definition, Preparation, Design and Finalization. If used effectively, JAD can accelerate design, increase quality and reduce development cost.

Stop to consider
JAD sessions are intended to pool in the subject matter expert's, the business analyst or the IT specialist in order to bring forth solutions.

2.5.3 Participatory Design

Participatory Design or PD is a useful alternative methodology to SDLC.PD puts a lot of emphasis on the role of the user than any other techniques does. In certain cases, PD may engage the entire end-user community in the process of development. It assumes that each user has equal share in ascertaining the system requirements and also in approving the system

design. In some other cases, PD allows an elected user group to control the development process. These user groups are representative of a larger community. Under Participatory Design, systems analysts work only for the users. The outside consultants as well as the organization's management give advice only rather than controlling it. Participatory Design, to some extent, is the outcome of the role of management and labor in workplace where the labor is more structured and is more closely in touch with the technological changes.

Stop to consider
PD is an important approach that puts emphasis on engaging the user in the development process

Check Your Progress
1. Under which conditions you would use the Prototype model .Explain 2. Bring out the basic difference between PD and JAD

This unit discussed about the fundamental framework that guides the systems analysis and design process, the SDLC or systems development life cycle, comprising of s seven major phases: the first being the project identification and selection, followed by project initiation and planning, then analysis, then logical design, then physical design, then implementation, and at the last maintenance. The life cycle has had its share of criticism too as no definite sequence can be followed. An urge for improvement and also to address the problems of SDLC led to the development many new alternative methodologies. These alternative mythologies include: Prototyping, the Joint Application Design and the Participatory Design. This unit concludes with an introduction to these approaches.

2.7 References and Suggested readings:

1) Systems Analysis And Design by Elias M.Awad; Galgotia Publications Pvt. Ltd.; Second Edition (1 January 2010)
2) Systems Analysis and Design by Kendall & Kendall; PHI; Fifth Edition.
3) System analysis and design methods by Jeffrey L. Whitten, Lonnie D. Bentley, Kevin C. Dittman;;Tata McGraw-Hill; Fifth Edition; 2001.
4)http://www.rspa.com

5) http://www.ieee.org

2.8 Model Question

A. Choose the correct option from the following:

1. How many steps are in the systems development life cycle (SDLC)?
   a) 1
   b) 4
   c) 7
   d) 8
2. Actual programming of software code is done during the _____ step in the SDLC.
   a).Maintenance
   b).Design

c).Analysis

d). Implementation

3. Enhancements, upgrades, and bug fixes are done during the _____ step in the SDLC.

a) Maintenance

b).Problem Identification

c). Design

d). Development and Documentation

4. The _____ determines whether the project should go forward.

a). feasibility assessment

b) Problem Identification

c). System Testing

d). program specification

5. RAD stands for

a). Rapid Application Development

b). Required Application Development

c). Rapid Application Developers

d). Rapid Application Disposition

B) Fill in the blanks:

1. Designing and building a scaled down version of a desired system is known _____.

2. The different types of feasibility are _____.

3._____ emphasizes on the role of user in system design.

4. Implementation includes _____

5. Prototyping is a form of _____

C) Descriptive Question:

1. What is System Development Life Cycle.Explain the steps in SDLC.

2. What is the difference between analysis and design? Explain

3. What are the basic types of feasibilities used in system analysis and design?

4. "Implementation of system can be done in three different ways". Do you agree with this statement? Justify your answer.

5. Describe a few methodologies used for system development.

2.9 Answers to questions

A) Multiple choice questions

1. (c)
2. (b)
3. (a)
4. (a)
5. (a)

B)

1. Prototyping
2. Corrective, adaptive and perfective.
3. Participatory Design
4. Coding, testing and installation
5. Rapid Application Development

BLOCK IV : UNIT-III
SYSTEM ANALYSIS

Unit Structure:

3.1 Introduction

In this unit, you will learn the process of planning the development of systems. You will also learn various techniques used in it. Here, you will learn about the various techniques of fact finding and getting appropriate information about system which is currently in place. Based on the information gathered, a list of requirements has been compiled. The same is sent to the customer for his/her review and comments. The topic of feasibility study is discussed in depth in this unit. The process of cost benefit analysis is also discussed in this unit. Lastly, we shall discuss about the tools used in structured analysis.

3.2 Objectives

After going through this unit, you will be able to learn:

- Know the various fact finding techniques and also their advantages and disadvantages.
- Identify check points in system life cycle to conduct feasibility study.
- Know the process of doing cost benefit analysis of the project.
- Know various issues related with system development.
- Learn the process of preparing schedule and
- Tools used in structured analysis.

3.3 Fact Finding Techniques

To learn the functions of the existing system, systems analyst needs to collect data related to the existing system. Usually, the data related to organization, staff, documents used, formats used in the input and output processes is collected. This information is obtained through interviews, group discussions, site visits, presentations, and questionnaires.

Need for Fact Finding

Normally, each and every business house or any organization has its own rules and procedures to run and manage it. When a system needs to be developed, the systems analyst needs to know the requirements of the system. Depending on these requirements, the system has to be developed.

3.3.1 Interviews

Personal interview is a recognized and most important fact finding technique, where the systems analyst gathers information from individual through face to face interaction. Interviews are used to find the facts, verify facts, clarify facts, get the customer involved, identify the system requirements and know all options. The interview is usually conducted by the systems analyst. To conduct interview, the interviewer must have personality which helps him/her to be social with strangers or different types of people. Always and for all situations, interviews are not appropriate fact finding methods. It has both advantages and disadvantages.

Advantages

- Interviews permit the systems analyst to get individual's views and get the specific problem work wise and operation wise.
- Interviews allow the systems analyst to obtain a better clarity of the problem due to feedback from the interviewees.
- In the process of interviews, the interviewer has time and scope to motivate the interviewee to respond freely and openly.
- Interviews allow the systems analyst to understand the user requirements and to know the problems faced by the user with the current system.
- It is an effective technique to gather information about complex existing systems.

Disadvantages

- Interviews are very time consuming.
- Success of interviews, in most of the cases, depends on the systems analyst's interpersonal relationship skills.
- Sometimes, interviews may be impractical due to the location of interviewees.

Types of Interviews: There are two types of interviews:

- **Structured Interviews:** There is a specific set of questions to be asked to an interviewee. In the case of unstructured interviews, there are few specific questions pertaining to an interviewee. But, you have questions which are common to all interviewees.
- **Unstructured Interviews:** Unstructured interviews are conducted with only a general goal or subject in mind.

Conducting Interview

Conducting Interview is an art. The success in interview depends on selecting the individual, preparing for the interview, creating situation in which the answers offered are reliable and creating a situation in which opinion can be given without any fear of being criticized by others.

Arranging Interview

The system analyst should prepare properly for the interview. S/he should select place of interview, time of interview in such a way so that there will be minimal interruption. Always, it is important to take appointment with the interviewee. Time to be spent during interview varies from project to project. The higher the management level of the interviewee, the less the time to be scheduled for the interview.

Guidelines for conducting interviews: For a successful interview, the steps to be followed are given below:

During introduction, the analyst should introduce himself by focusing on purpose of the interview and the confidential nature of interview. Also, this is the phase wherein first impressions are formed and pave way for the success of the remaining part of the interview.

- **Asking questions**
  Questions should be asked exactly as these are worded in case of structured interview. Rewording may modify or bias the response. Always, questions have to asked in the same sequence as prepared.
- **Recording the interview**
  Record of the interview must be kept mentioning the source of the data and its time of collection. Sometimes, the analyst cannot remember the source of the data which may attribute to the invalid sources.
- **Doing a final check**
  After the interview has been completed, the deliberations made during the interview should be put in the form of a report. The report of the interview has to be sent to the

interviewee for his/her signature. If any discrepancies are found or any modifications are to be done, these can be done at this point of time.

3.3.2. Group Discussion

In this method, a group of staff members are invited who are expected to be well versed in their own wings of the organization. The analysts will have a discussion with the members for their views and responses to various queries posed by them. In this process, individuals from different sections gather together and will discuss the problem at hand. Ultimately, they come to an optimum solution. In this process, the problems of all sections are taken care of most of the cases, solutions are found which are acceptable to everyone. The main disadvantage of this process is that it is very difficult to get all the concerned people together at a time. But, the major advantage is that a mutually acceptable solution can be found.

3.3.3. Site Visits

The engineers of the development organization visit the sites. Usually, the systems analysts visit sites to get first hand information of the working of the system. In this technique, systems analyst watches the activities of different staff members to learn about the system. When there is confusion about the validity of data collected from other sources, the systems analyst uses the method of site visits. The main objective of site visit is to examine the existing system closely and record the activities of the system.

Advantages:

- The process of recording facts site visits is highly reliable.
- Sometimes, site visits take place to clear doubts and check the validity of the data.
- Site visit is inexpensive when compared to other fact finding techniques.
- In this technique, systems analyst will be able to see the processes in the organization at first hand.
- The systems analyst can easily understand the complex processes in the organization.

Disadvantages:

- People usually feel uncomfortable when being watched; they may unwillingly perform their work differently when being observed.
- Due to interruptions in the task being observed, the information that is collected may be inaccurate.
- Site visits are done during a specific period and during that period, complexities existing in the system may not be experienced.
- There may be scheduling problems for the systems analysts when the activities take place during odd hours.
- Sometimes, people may be more careful to adopt the exact procedure which they do not typically follow.

Guidelines for site visit

Site visits are to be conducted where the work load is normal. After studying the work and normal work load, systems analyst can observe the work at peak hours to see the effect caused by increased volumes. The systems analyst should collect the input /output form,

documents at the time of his/her visit. The following guidelines need to be followed at the time of observation and site visit:

- Keep a low profile at the time of site visit.
- Take necessary permissions from appropriate officials to conduct site visit.
- Inform the individuals who will be observed at the time of site visit.
- Take notes of the study of site visit immediately.
- Do not make any assumptions.

3.3.4 Presentations

It is another way of finding the facts and collecting data. Presentation is the way by which the systems analyst gathers firsthand knowledge of the project. The customer makes a presentation of the existing system or about the organization. Participants in the meeting are representatives from the IT Company and key personnel of the client organization. When a company needs to develop a software project, it may present its requirements for IOE (interest of expression) from the interested IT Company. In that case, the client presents his/her requirements. Based on the requirements, the IT companies make prototype and show the demo of the prototype. It is very difficult to obtain information in detail from a presentation. But, information available through presentation is sufficient to develop a prototype. Presentation is made by the concerned department in consultation from other departments and senior officials.

3.3.5 Questionnaires

Questionnaires are special purpose documents that allow the analyst to collect information and opinion from respondents. By using questionnaires, it is possible to collect responses or opinion from a large number of people. This is the only way to get response from a large audience.

Advantages:

- It is an inexpensive means of collecting the data from a large group of individuals.
- It requires less skill and experience to administer questionnaires.
- Proper formulation and interaction with respondents leads to unbiased response from the customers.
- Customers can complete it at their convenience.
- Responses can be tabulated and analyzed quickly.

Disadvantages:

- Sometimes, the number of respondents is low.
- There is no guarantee that the respondents will answer all the questions.
- Sometimes, the individual may misunderstand the question. In that situation, the analyst may not get correct answer.

Types of questionnaires

- ***Free formed questionnaires*** are questionnaires where questions are mentioned along with blank spaces for response.
- ***Fixed formed questionnaires*** are questionnaires which consist of multiple choices and the respondent can select only from the choices provided.

The following are various types of fixed format questions:

- True / false or yes/no type questions.
- Questions whose response will be one of the choices: strongly agree, agree, and disagree.
- Ranking type questions (ranking items in order of importance).
- Multiple choice questions (select one response or all the relevant responses).

Check Your Progress 1

1. …… and ……..…… are two types of interviews.
2. …… are special purpose documents that allow the analyst to collect information and opinion from respondents.
3. The engineers of the …..…… organizations make site visits.

3.4 Issues Involved in Feasibility Study

Feasibility study consists of activities which determine the existence of scope of developing an information system to the organization. This study should be done throughout the life cycle. In a project, at one point of time, it may seem that the project is feasible. But, after proceeding one or two phases, it may become infeasible. So, it is necessary to evaluate the feasibility of a project at the earliest possible time. Months or years of efforts, huge finances could be saved if an infeasible system is recognized at earlier stage.

Feasibility study starts from the preliminary investigation phase. At this stage, the analyst estimates the urgency of the project and estimates the development cost.

The next check point is problem analysis. At this stage, the analyst studies current system. S/he does it to understand the problem in the better way. It helps him/her to make better estimates of development cost, and also to find out the benefits to be obtained from the new system. In feasibility analysis, we have to study the following:

- Technical feasibility,
- Operational feasibility,
- Economic feasibility, and
- Legal Feasibility.

3.4.1 Technical Feasibility

Technical feasibility is concerned with the availability of hardware and software required for the development of the system, to see compatibility and maturity of the technology proposed to be used and to see the availability of the required technical manpower to develop the system. These three issues are addressed during this study.

Is the proposed technology proven and practical? At this stage, the analyst has to see or identify the proposed technology, its maturity, its ability or scope of solving the problem. If the technology is mature, if it has large customer base, it will be preferable to use as large customer base already exists and problems that stem from its usage may be less when compared to other technologies which don't

have a significant customer base. Some companies want to use the state of art new technology irrespective of the size of customer base.

The next question is: does the firm possess the necessary technology it needs. Here, we have to ensure that the required technology is practical, and available. Now, does it have the required hardware, and software? For example, we need ERP software, and hardware which can support ERP. Now, if our answer is no for either of the questions, then the possibility of acquiring the technology should be explored.

The last issue related to technical feasibility is the availability of technical expertise. In this case, Software and Hardware are available. But, it may be difficult to find skilled manpower. The company might be equipped with ERP software, but the existing manpower might not have the expertise in it. So, the manpower should be trained in the ERP software. This may lead to slippage in the delivery schedules.

3.4.2 Operational Feasibility

Operational feasibility is all about problems that may arise during operations. There are two aspects related with this issue:

- What is the probability that the solution developed may not be put to use or may not work?
- What is the inclination of the management and end users towards the solution? Though, there is very least possibility of management being averse to the solution, there is a significant probability that the end users may not be interested in using the solution due to lack of training, insight, etc.

Also, there are other issues related with operational feasibility.

- **Information:** The system needs to provide adequate, timely, accurate and useful information. It should be able to supply all the useful and required information to all levels and categories of users.
- **Response time:** It needs to study the response time of the system in term of throughput. It should be fast enough to give the required output to the users.
- **Accuracy:** A software system must operate accurately. It means that it should provide value to its users. Accuracy is the degree to which the software performs its required functions and gives desired output correctly.
- **Security:** There should be adequate security to information and data. It should be able to protect itself from fraud.
- **Services:** The system needs to be able to provide desirable and reliable services to its users.
- **Efficiency:** The system needs to be able to use maximum of the available resources in an efficient manner so that there are no delays in execution of jobs.

3.4.3 Economic Feasibility

It is the measure of cost effectiveness of the project. The economic feasibility is nothing but judging whether the possible benefit of solving the problems is worthwhile or not. At the feasibility study level, it is impossible to estimate the cost because customer's requirements and alternative solutions have not been identified at this stage. However, when the specific

requirements and solutions have been identified, the analyst weighs the cost and benefits of all solutions, this is called "cost benefit analysis". This is discussed below. A project which is expensive when compared to the savings that can be made from its usage, then this project may be treated as economically infeasible.

3.4.4 Legal Feasibility

Legal feasibility studies issues arising out of the need to the development of the system. The possible consideration might include copyright law, labour law, antitrust legislation, foreign trade, regulation, etc. Contractual obligation may include the number of users who will be able to use the software. There may be multiple users' licences, single user licences, etc. Legal feasibility plays a major role in formulating contracts between vendors and users. If the ownership of the code is not given to the user, it will be difficult to install it without proper permission to other systems. Another important legal aspect is that whenever an IT company and the user company do not belong to the same country then the tax laws, foreign currency transfer regulations, etc., have to be taken care of.

3.5 Cost Benefit Analysis

In economic feasibility, cost benefit analysis will be done. There are two types of costs associated with a project: The costs involved with development of the system and costs associated with operation and maintenance of the system. System development cost can be estimated at the time of planning of the system and it should be refined in different phases of the project. Maintenance and operation costs are to be estimated beforehand. At the same time, these estimations are bound to change as the requirements change during the development process. After the implementation, these costs may increase or decrease depending on the nature of updating done to the system. System development cost is one time cost, but maintenance and operating costs are recurring costs. Different costs are:

- ***Cost of human resources*:** It includes the salaries of system analysts, software engineers, programmers, data entry operators, operational, and clerical staff. In other words, the amount that is going to be spent on all the people involved.
- ***Cost of infrastructure*:** The cost of infrastructure including those of computers, cables, software, etc., comes under this head.
- ***Cost of training*:** Both the developing staff and operating staff need to be trained for new technologies and new system. So, the training cost has to be considered for calculating the cost of the system.

There are two components in economic feasibility: **costs and benefits**. The cost consists of tangible hardware, software costs, cost of human resources and some intangible costs. Tangible costs are saved by the usage of the system. Intangible costs are saved by the quality of the system. Also, application of system should lead to efficiency. When the quality of the system is high, the effectiveness of the services provided by the organizations increase. If a choice has to be made between efficiency and effectiveness then it is better to do the right thing inefficiently than to do wrong thing efficiently. The tangible benefits are those which can be quantified easily. They can be measured in terms of savings or profits. On the other hand, in the case of intangible benefits, it is difficult to quantify. Examples of intangible

benefits are improving company goodwill, improving employee moral, better decision making, etc.

We may consider the case of an insurance company's branch office. There are 15 employees in the office which include one manager, two business development officers, one accounts officer, one administrative officer, seven clerical staffs, two security guards, one peon. If the branch is converted to a fully computerized branch, the total hardware and software cost will be Rs.10 lakhs. Training of the existing manpower will be Rs.50, 000. Total investment is 10.5 lakhs. Total maintenance cost of software and hardware is Rs.1.5 lakhs per year. Interest of the investment is Rs.1, 26,000 per year. So, the total expenditure is increased by Rs.2, 76,000 per year. But the branch can reduce the clerical staff. Now it needs two clerical staffs and two data entry operators. Total cost saving by reducing 3 staffs will be Rs.4.5 lakhs per year. Here, it is clear that the tangible benefit is more than the expenditure. Besides, the tangible benefit also improves the customer's satisfaction. So, it is clear that the project is feasible.

Check Your Progress 2

1. …….... consists of activities which determine the existence of scope of developing an information system to the organization.
2. ……..……. is all about problems that may arise during operations.
3. …… and ……..…… are two components of economic feasibility.

3.6 Preparing Schedule

A system development process scheduling is an activity that distributes estimated effort according to the planned project duration by allocating the effort to specific software engineering tasks. But, at the early stage of the project, macroscopic schedule is developed. This schedule identifies all major activities of the project. As the project progresses, each entity of macroscopic schedule is refined into a detailed schedule. For a systems development, scheduling is meant for setting an end date to the project(s).

Now, the feasibility of following the schedule is directly related to the time table made. Systems analysts have to take care of schedule feasibility of the system. The purpose of schedule feasibility is to understand the time frames and dates of completion of different phases of the project. It means that the project can be completed and be operational so that it will meet the needs of the user requirements.

In most cases, missing the deadline may invite penalties. A systems analyst has to remember the schedule feasibility at the time before entering into any agreement with client regarding the delivery schedules. At the project planning stage, feasibility of conforming to the schedule will be studied by the analyst. To take a decision, factors such as expected team size, availability of resources, sub-contracting or outsourcing of activities have to be considered. Scheduling feasibility will be reassessed during the commencement of each phase.

3.7 Structured Analysis

Structured Analysis is a development method that allows the analyst to understand the system and its activities in a logical way. It is a systematic approach, which uses graphical tools that analyze and refine the objectives of an existing system and develop a new system specification which can be easily understandable by user. It has following attributes:

- It is graphic which specifies the presentation of application.
- It divides the processes so that it gives a clear picture of system flow.
- It is logical rather than physical i.e., the elements of system do not depend on vendor or hardware.
- It is an approach that works from high-level overviews to lower-level details.

3.8. Tools used in Structured Analysis

During Structured Analysis, various tools and techniques are used for system development. They are −

- Data Flow Diagrams
- Data Dictionary
- Decision Trees
- Decision Tables
- Structured English
- Pseudo Code

3.8.1 Data Flow Diagram

It is a technique developed by Larry Constantine to express the requirements of system in a graphical form.

- It shows the flow of data between various functions of system and specifies how the current system is implemented.
- It is an initial stage of design phase that functionally divides the requirement specifications down to the lowest level of detail.
- Its graphical nature makes it a good communication tool between user and analyst or analyst and system designer.
- It gives an overview of what data a system processes, what transformations are performed, what data are stored, what results are produced and where they flow.

Basic Elements of DFD

DFD is easy to understand and quite effective when the required design is not clear and the user want a notational language for communication. However, it requires a large number of iterations for obtaining the most accurate and complete solution.

The following table shows the symbols used in designing a DFD and their significance:

Symbol Name	Symbol	Meaning
Square		Source or Destination of Data
Arrow		Data Flow

Circle		Process transforming data flow
Open Rectangle		Data Store

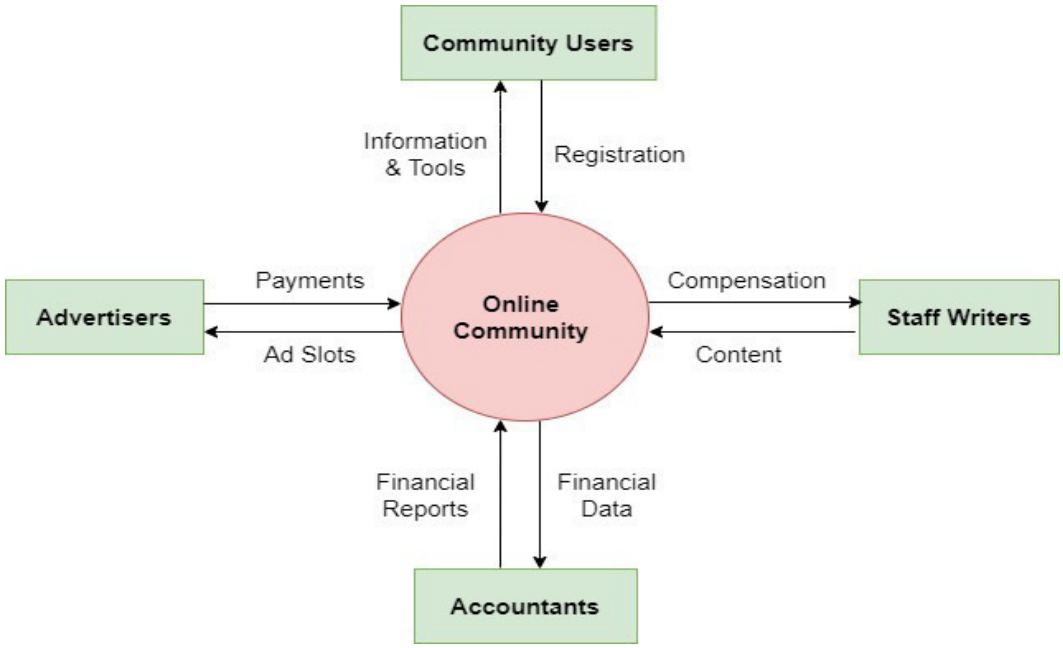
Types of DFD

DFDs are of two types: Physical DFD and Logical DFD. The following table lists the points that differentiate a physical DFD from a logical DFD.

Physical DFD	Logical DFD
It is implementation dependent. It shows which functions are performed.	It is implementation independent. It focuses only on the flow of data between processes.
It provides low level details of hardware, software, files, and people.	It explains events of systems and data required by each event.
It depicts how the current system operates and how a system will be implemented.	It shows how business operates; not how the system can be implemented.

Context Diagram

A context diagram helps in understanding the entire system by one DFD which gives the overview of a system. It starts with mentioning major processes with little details and then goes onto giving more details of the processes with the top-down approach. The context diagram of mess management is shown below.



3.8.2 Data Dictionary

A data dictionary is a structured repository of data elements in the system. It stores the descriptions of all DFD data elements that is, details and definitions of data flows, data stores, data stored in data stores, and the processes.

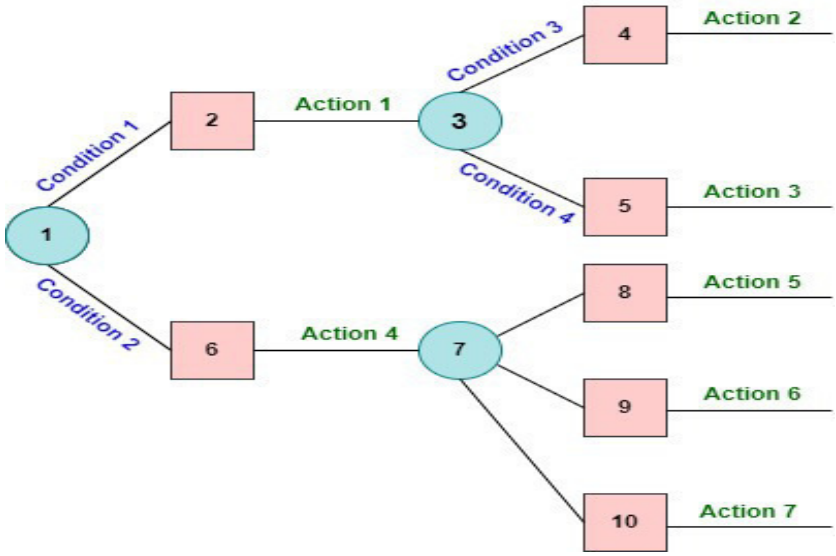
A data dictionary improves the communication between the analyst and the user. It plays an important role in building a database. Most DBMSs have a data dictionary as a standard feature. For example, refer the following table:

Sl. No.	Data Name	Description	No. of Characters
1	ISBN	ISBN Number	10
2	TITLE	Title	60
3	SUB	Book Subjects	80
4	ANAME	Author Name	15

3.8.3 Decision Trees

Decision trees are a method for defining complex relationships by describing decisions and avoiding the problems in communication. A decision tree is a diagram that shows alternative actions and conditions within horizontal tree framework. Thus, it depicts which conditions to consider first, second, and so on.

Decision trees depict the relationship of each condition and their permissible actions. A square node indicates an action and a circle indicates a condition. It forces analysts to consider the sequence of decisions and identifies the actual decision that must be made.



3.8.4 Decision Tables

Decision tables are a method of describing the complex logical relationship in a precise manner which is easily understandable.

- It is useful in situations where the resulting actions depend on the occurrence of one or several combinations of independent conditions.
- It is a matrix containing row or columns for defining a problem and the actions.

Components of a Decision Table

- **Condition Stub**: It is in the upper left quadrant which lists all the condition to be checked.
- **Action Stub**: It is in the lower left quadrant which outlines all the action to be carried out to meet such condition.
- **Condition Entry**: It is in upper right quadrant which provides answers to questions asked in condition stub quadrant.
- **Action Entry**: It is in lower right quadrant which indicates the appropriate action resulting from the answers to the conditions in the condition entry quadrant.

The entries in decision table are given by Decision Rules which define the relationships between combinations of conditions and courses of action. In rules section,

- Y shows the existence of a condition.
- N represents the condition, which is not satisfied.
- A blank - against action states it is to be ignored.
- X (or a check mark will do) against action states it is to be carried out.

For example, refer the following table:

CONDITIONS	**Rule 1**	**Rule 2**	**Rule 3**	**Rule 4**
Advance payment made	Y	N	N	N
Purchase amount = Rs. 10,000/-	-	Y	Y	N
Regular Customer	-	Y	N	-
ACTIONS				
Give 5% discount	X	X	-	-
Give no discount	-	-	X	X

3.8.5 Structured English

Structure English is derived from structured programming language which gives more understandable and precise description of process. It is based on procedural logic that uses construction and imperative sentences designed to perform operation for action.

- It is best used when sequences and loops in a program must be considered and the problem needs sequences of actions with decisions.
- It does not have strict syntax rule. It expresses all logic in terms of sequential decision structures and iterations.

For example, see the following sequence of actions:

```
if customer pays advance
  then
    Given 5 % Discount
  else
    if purchase amount >=10,000
      then
        if the customer is a regular customer
        then Give 5 % Discount
        else No Discount
        endif
    endif
endif
```

3.8.6 Pseudo Code

A pseudo code does not conform to any programming language and expresses logic in plain English.

- It may specify the physical programming logic without actual coding during and after the physical design.
- It is used in conjunction with structured programming.
- It replaces the flowcharts of a program.

Guidelines for Selecting Appropriate Tools

Use the following guidelines for selecting the most appropriate tool that would suit your requirements −

- Use DFD at high or low level analysis for providing good system documentations.
- Use data dictionary to simplify the structure for meeting the data requirement of the system.
- Use structured English if there are many loops and actions are complex.
- Use decision tables when there are a large number of conditions to check and logic is complex.
- Use decision trees when sequencing of conditions is important and if there are few conditions to be tested.

3.9 Let us Sum Up

The process of systems planning is a critical activity in the life of a project. Here, we have focused on determination of requirements, gathering of information about the existing system. There are many techniques for requirements determination which include interviews, questionnaires, group discussions, site visits, and presentations. One or more of the above techniques are used to gather adequate information about the current system. Each technique has its own advantages and disadvantages. In personal interview, the systems analyst gathers information through face to face interaction. It is very common and simple method of fact finding. In a group discussion, a group of individuals is called from different work groups. In

this method, problems of all the sections are discussed and a suitable and acceptable solution is arrived at. In the process of site visits, the systems analyst watches the activities and learns about the system. Questionnaires are special type of documents which allow the system analyst to collect information from the respondent.

In this unit, the process of study of feasibility of developing the system is examined. In feasibility study, it is stated whether the project assessment can be accepted for development or is to be rejected for its infeasibility. The key activity in the project planning is the assessment of different feasibility issues associated with the project. It includes economic, technical, operational and legal issues. The economic feasibility judges the cost effectiveness of the project. There are two types of costs involved in a project:

- System Development Costs.
- Operation and Maintenance Costs.

The benefit consists of saving the tangible costs by using the system and the intangible costs by improving the quality of service. In operational feasibility, systems analyst assesses the degree to which the proposed system solves business problem or takes advantage of business opportunity. The legal issues to be considered are copyright law, antitrust legislation, foreign trade legislation, etc.

There are several structural analysis tools that is described in this unit, and analysts work together through a series of meetings to specify system requirements.

3.10 Answers to Check Your Progress

Check Progress 1:

1. Structured interviews, Unstructured interviews.
2. Questionnaires.
3. Development.

Check Progress 2:

1. Feasibility Study.
2. Operational Feasibility.
3. Costs and benefits.

3.11 Further Readings

1. Jeffrey A.Hoffer, Joey F.George and Joseph S.Valacich; Modern Systems Analysis and Design; Pearson Education; Third Edition; 2002.
2. Jeffrey L. Whitten, Lonnie D. Bentley, Kevin C. Dittman; Systems Analysis and Design Methods; Tata McGraw Hill; Fifth Edition; 2001.
3. Elias M. Awad; Systems Analysis and Design; Galgotia Publications; Second Edition; 1994.
4. Perry Edwards; Systems Analysis and Design; McGraw Hill Publication;1993.

3.12 Probable Questions

1. Explain the fact finding techniques in details.
2. Describe the feasibility study with respect to system analysis.

3. What is cost-benefit analysis?
4. What is scheduling?
5. Define structure analysis? What are the tools used in structured analysis?
6. Explain DFD with suitable example.
7. Differentiate between physical DFD and logical DFD.
8. Define the following terms:
    a) Decision Trees
    b) Decision Tables
    c) Structured Query
    d) Data Dictionary.

BLOCK IV : UNIT-IV
SYSTEM DESIGN

Unit Structure:

4.1 INTRODUCTION TO SYSTEM DESIGN

It is the process of specifying the components or modules of a new business system or replacing an existing system in order to meet certain needs. Before you begin planning, you

must first completely understand the old system and identify how computers may be employed most effectively in order to run efficiently. The goal of system design is to figure out how to achieve the system's goal. The main focus of System Analysis and Design (SAD) is on Systems, Processes, and Technology. You will learn design principles such as top down design and bottom up design in this unit. Also, describe the steps of system design, such as the analysis and design phases. This lesson also covers several design approaches such as Data Flow Diagrams, E-R Diagrams, Decision Tables, Decision Trees, and Structured English notation. This unit discusses the design of forms and reports, as well as the importance of forms and reports, as well as the contrasts between them. This section also covers criteria for designing forms and reports, as well as deliverables and outcomes. You will learn how to design physical files and databases in this class. The process of designing database fields, physical records, and files is also covered in this unit.

4.2 OBJECTIVES

After going through this unit, you will be able to learn:

- Know the meaning of design
- Learn the process of top down design and bottom up design
- Learn the stages of design
- Different methodologies of system design
- Define Forms & Reports and their importance in real life
- List the process of designing Forms & Reports
- Apply the general guidelines for formatting Forms and Reports
- Specify different criteria for designing Forms and Reports
- Advantage of databases over files
- Difference between logical and physical design
- Rules for good database design practices
- Define the concepts of fields, records and database
- How to design the fields and records in a database table
- Understand various constrains enforced during database design
- Typical database design concepts and
- Explain the concepts of records, record types, and files.

4.3 DESIGN PRINCIPLES

There are certain principles that can be used for the development of the system. These principles are meant to effectively handle the complexity of process of design. These principles are:

Problem Partitioning: It is concerned with partitioning the large problems. *Divide and Conquer* is the policy adopted here. The system is divided into modules that are self dependent. It improves the efficiency of the system. It is necessary that all modules have interaction between them.

Abstraction: It is an essential component of the design process and is required for problem splitting. Abstraction is a tool that allows a designer to consider a component on an abstract level (outer view) without having to worry about the component's implementation specifics.

When a challenge is broken down into smaller pieces, abstraction is required so that one design process can be completed successfully and efficiently. Abstraction can be either functional or data-related. We specify the module by the function it performs in functional abstraction. Data is buried behind functions/operations in data abstraction. Object-oriented design is built on the foundation of data abstraction.

For effective software design, design concepts are required. Top down and bottom up methods help in the implementation of these principles and the achievement of the goals. A system is made up of components known as modules, each of which has subordinate modules. A system is made up of components in a hierarchy, with the total system being the highest-level module, known as the super ordinate module. There are two techniques to designing a hierarchy like this: top down and bottom up. The top-down approach begins with the highest-level module in the hierarchy and works its way down. The bottom-up technique, on the other hand, begins with lower-level modules and progresses through higher levels to the top-level module.

4.3.1 Top-down Design Process

This approach starts by identifying major components of the system decomposing them into their own subordinate level components and interacting until the desired level of detail is achieved. Top down design methods often result in some form of stepwise refinement, starting from an abstract design, in each step, the design is refined to a more concrete level until we reach a level where no more refinement is required and the design can be implemented directly. This approach is explained by taking an example of "Library Information System" depicted in figures 1, 2 and 3 below:

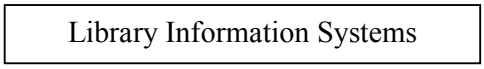


Figure 1: The top (root) of software system

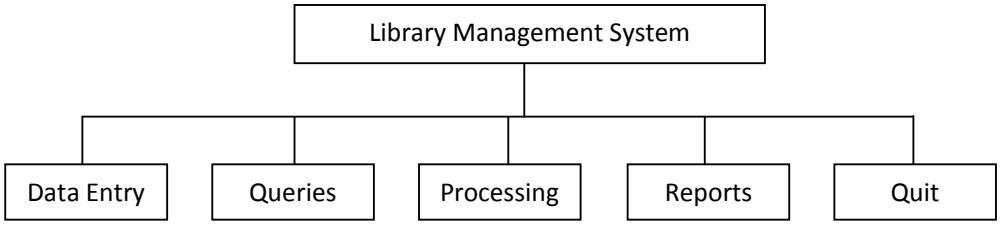
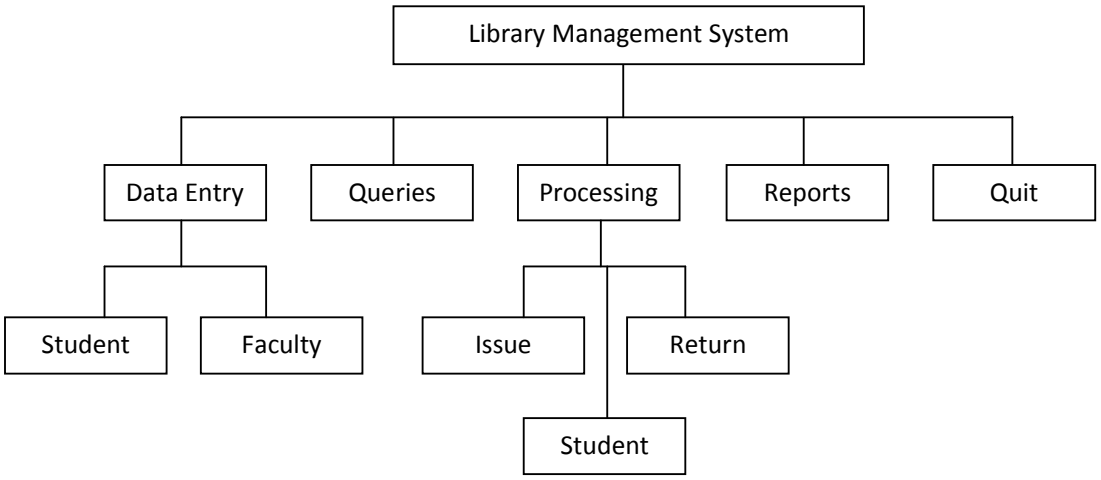


Figure 2: Further decomposition of the "top" of software system

Now, we can move further down and divide the system even further as shown in **Figure 3.**



4.3.2 Bottom-up Design Process

In bottom up strategy, we start from the bottom and move upwards towards the top of the software. This approach leads to a style of design where we decide the process of combining modules to provide larger ones, to combine these to provide even larger ones and so on till we arrive at one big module. That is, the whole of the desired program. This method has one weakness. We need to use a lot of intuition to decide the functionality that is to be provided by the module. If a system is to be built from existing system, this approach is more suitable as it starts from some existing modules.

4.4 STAGES OF SYSTEM DESIGN

4.4.1 Analysis Phase: Analysis Phase involves data flow diagram, data dictionary, state transition diagram, and entity-relationship diagram.

1. **Data Flow Diagram:** In the data flow diagram, the model describes how the data flows through the system. We can incorporate the Boolean operators and & or link data flow when more than one data flow may be input or output from a process. For example, if we have to choose between two paths of a process we can add an operator or and if two data flows are necessary for a process we can add an operator. The input of the process “check-order” needs the credit information and order information whereas the output of the process would be a cash-order or a good-credit-order.
2. **Data Dictionary:** The content that is not described in the DFD is described in the data dictionary. It defines the data store and relevant meaning. A physical data dictionary for data elements that flow between processes, between entities, and between processes and entities may be included. This would also include descriptions of data elements that flow external to the data stores. A logical data dictionary may also be included for each such data element. All system names, whether they are

names of entities, types, relations, attributes, or services, should be entered in the dictionary.

3. **State Transition Diagram:** State transition diagram is similar to the dynamic model. It specifies how much time the function will take to execute and data access triggered by events. It also describes all of the states that an object can have, the events under which an object changes state, the conditions that must be fulfilled before the transition will occur and the activities were undertaken during the life of an object.
4. **ER Diagram:** ER diagram specifies the relationship between data store. It is basically used in database design. It basically describes the relationship between different entities.

4.4.2 Design Phase: Design Phase involves structure chart and pseudocode.

1. **Structure Chart:** It is created by the data flow diagram. Structure Chart specifies how DFS's processes are grouped into tasks and allocate to the CPU. The structured chart does not show the working and internal structure of the processes or modules and does not show the relationship between data or data-flows. Similar to other SASD tools, it is time and cost-independent and there is no error-checking technique associated with this tool. The modules of a structured chart are arranged arbitrarily and any process from a DFD can be chosen as the central transform depending on the analysts' own perception. The structured chart is difficult to amend, verify, maintain, and check for completeness and consistency.
2. **Pseudo Code:** It is the actual implementation of the system. It is an informal way of programming that doesn't require any specific programming language or technology.

Check Your Progress 1

1. ……..…… is a tool that permits the designer to consider a component at an abstract level (outer view) without worrying about details of implementation of the component.
2. ……..… starts by identifying major components of the system decomposing them into their own subordinate level components and interacting until the desired level of detail is achieved.
3. …… starts from the bottom and move upwards towards the top of the software.
4. A ……..…… depicts the modular organization of an information system.
5. If a menu based system is concerned, the main menu may be considered as the …… and the menu options in it may be considered as subordinate modules.
6. Modules at …….. don't call any other modules.

4.5 LOGICAL AND PHYSICAL DESIGN

Logical Design

Logical design pertains to an abstract representation of the data flow, inputs, and outputs of the system. It describes the inputs (sources), outputs (destinations), databases (data stores), procedures (data flows) all in a format that meets the user requirements. While preparing the logical design of a system, the system analyst specifies the user needs at level of detail that virtually determines the information flow into and out of the system and the required data sources. Data flow diagram, E-R diagram modeling are used. It is the phase of system development life cycle in which system analyst and user develops concrete understanding of the operation of the system. **Figure 4** depicts various steps involved in the logical design. It includes the following steps:

- Designing forms (hard copy and computer displays) and reports, which describe how data will appear to users in system inputs and outputs;
- Designing interfaces and dialogues, which describe the pattern of interaction between users and software; and
- Designing logical databases, which describe a standard structure for the database of a system that is easy to implement in a variety of database technologies.

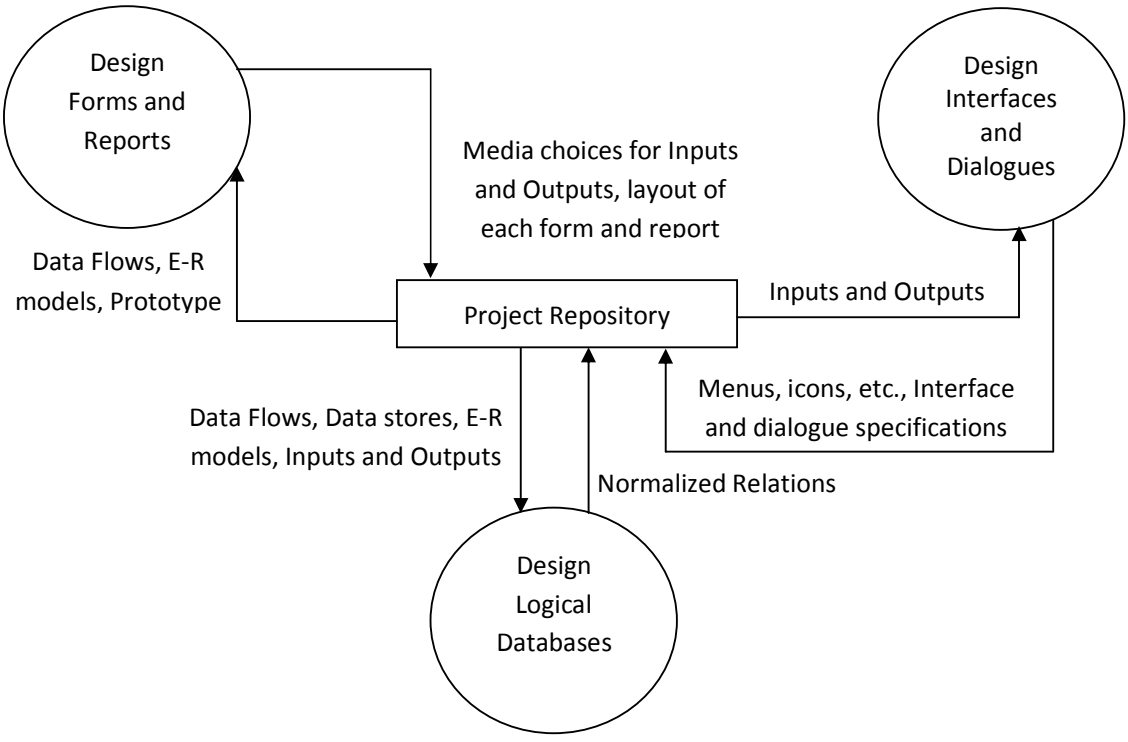


Figure 4: Steps in logical design

In logical design, all functional features of the system chosen for development in analysis are described independently of any computer platform. Logical design is tightly linked to previous system development phases, especially analysis. The three sub phases mentioned in

the *Figure 4* are not necessarily sequential. The project dictionary or CASE repository becomes an active and evolving component of system development management during logical design. The complete logical design must ensure that each logical design element is consistent with others and satisfactory to the end user.

User Interface Design

User interface design is concerned with the dialogue between a user and the computer. It is concerned with everything from starting the system or logging into the system to the eventually presentation of desired inputs and outputs. The overall flow of screens and messages is called a dialogue. The following are various guidelines for user interface design:

a) The system user should always be aware of what to do next.
b) The screen should be formatted so that various types of information, instructions and messages always appear in the same general display area.
c) Messages, instructions or information should be displayed long enough to allow the system user to read them.
d) Use display attributes sparingly.
e) Default values for fields and answers to be entered by the user should be specified.
f) A user should not be allowed to proceed without correcting an error.
g) The system user should never get an operating system message or fatal error.
h) If the user does something that could be catastrophic, the keyboard should be locked to prevent any further input, and an instruction to call the analyst or technical support should be displayed.

Logical Database Design

Data modeling is used for logical database design. A conceptual model of data used in an application is obtained by using an entity relationship model (E-R model). E-R model assists in designing relational databases. A relational database consists of a collection of relations relevant for a specified application. A relation is a table which depicts an entity set. Each column in the relation corresponds to an attribute of the entity. Each row contains a member of the entity set. Normalization is a procedure used to transform a set of relations into another set which has some desirable properties. Normalization ensures that data in the database are not unnecessarily duplicated. It also ensures that addition and deletion of entity rows (or tuples) or change of individual attribute values do not lead to accidental loss of data or errors in database.

Physical Design

Physical design relates to the actual input and output processes of the system. It focuses on how data is entered into a system, verified, processed, and displayed as output. It produces the working system by defining the design specification that specifies exactly what the candidate system does. It is concerned with user interface design, process design, and data design. It consists of the following steps:

- Specifying the input/output media, designing the database, and specifying backup procedures.
- Planning system implementation.

- Devising a test and implementation plan, and specifying any new hardware and software.
- Updating costs, benefits, conversion dates, and system constraints.

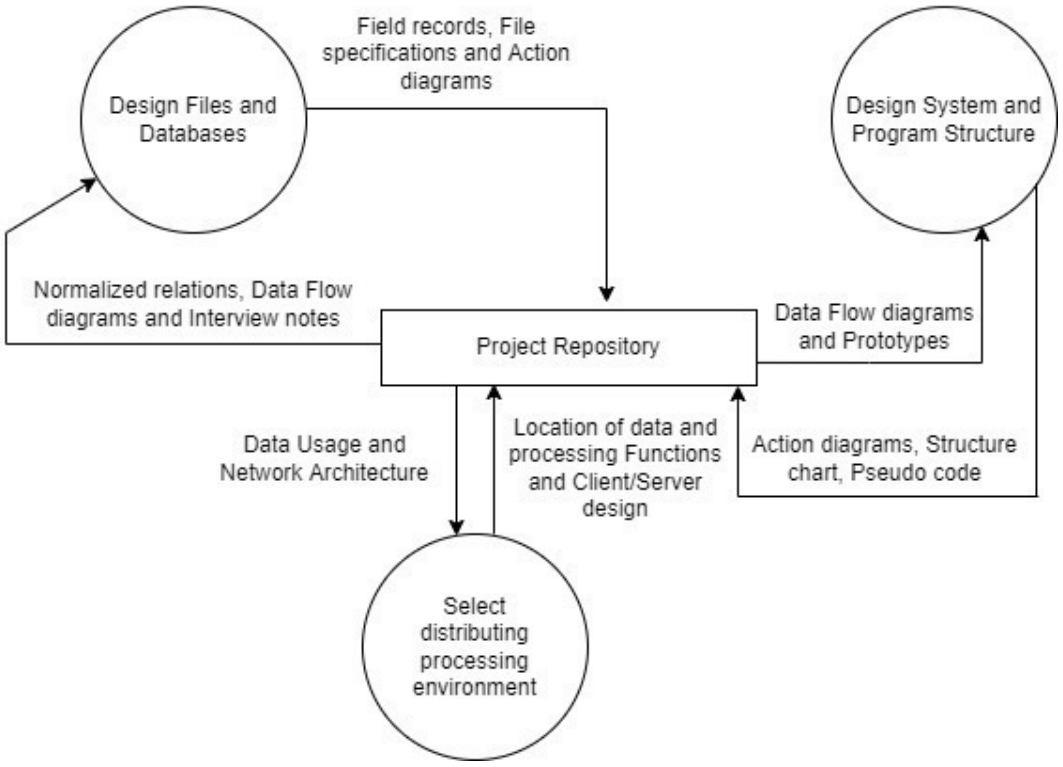


Figure 5 depicts various steps involved in physical design

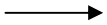
4.6 DESIGN METHODOLOGY

1) **Process Modeling:** It involves graphically representing the functions or processes, which capture, manipulate, store and distribute data between a system and its environment and between components within a system. A common form of a process model is called the **Data Flow Diagrams**. It represents the system overview. A DFD can be categorized in the following forms:
   a) **Context diagram**: An overview of an organizational system that shows the system boundaries, external entities that interact with the system and the major information flows between the entities and the system. In this diagram, a single process represents the whole system.
   b) **First level DFD**: A data flow diagram that represents a system's major processes, data flows, and data stores at a high level of detail.
   c) **Functional decomposition diagram**: Functional decomposition is an iterative process of breaking the description of a system down into finer and finer detail which creates a set of charts in which one process on a given chart is explained in greater detail on another chart. In this diagram, sub-processes of first level DFD are explained in detail.

There is no limit on the number of levels of Data Flow Diagrams that can be drawn. It depends on the project at hand. The following are various components of a Data Flow Diagram:

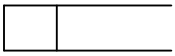
1. **Process:** During a process, the input data is acted upon by various instructions whose result is transformed data. The transformed data may be stored or distributed. When modeling the data processing of a system, it doesn't matter whether the process is performed manually or by a computer. People, procedures or devices can be used as processes that use or produce (transform) data. The notation for process is

2. **Data Flow:** Data moves in a specific direction from a point of origin to point of destination in the form of a document, letter, telephone call or virtually any other medium. The data flow is a "packet" of data. The notation for data flow is



3. **Source or sink of data:** The origin and /or destination of data sometimes referred to as external entities. These external entities may be people, programs, organization or other entities that interact with the system but are outside its boundaries. The term source and sink are interchangeable with origin and destination. The notation for source or sink is

4. **Data store:** A data store is data at rest, which may take the form of many different physical representations. They are referenced by a process in the system. The data store may reference computerized or non-computerized devices. Notation for data store is

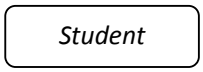


Rules for drawing a data flow diagram	
Components	**Rules**
For process	➢ No process can have only outputs. ➢ No process can have only inputs. ➢ A process has a verb phrase label.
For data store	➢ Data cannot move directly from one data store to another data store. Data must be moved through a process. ➢ Data cannot move directly from an outside source to data store. Data must be moved through a process that receives data from the source and places it into the data store. ➢ Data cannot move directly to an outside sink from a data store. Data must be moved through a process. ➢ A data store has a noun phrase label.

For source/sink	➢ Data cannot move directly from a source to a sink. It must be moved by a process ➢ A source/sink has a noun phrase label
For data flow	➢ A data flow has only one direction of flow between symbols. It may flow in both directions between a process and a data store to show a read operation before an update. ➢ A data flow cannot go directly back to the same process it leaves. There must be at least one other process which handles the data flow, produces some other data flow and returns the original data flow to the beginning process. ➢ A data flow to a data store means update (delete or change). ➢ A data flow from a data store means retrieve or use. ➢ A data flow has a noun phrase label.

Table 1 shows the rules for drawing a data flow diagram.

2) **Data Modeling:** It is a technique for organizing and documenting a system's data. Data modeling is sometimes called database modeling because a data model is eventually implemented as database. It is also sometimes called information modeling. The tool for data modeling is **Entity Relationship diagram** or **ER Diagram**. It depicts data in terms of entities and relationships described by the data. Martin gives the following notations for the components of ERD.
   a) **Entities:** An entity is something about which the business needs to store data. An entity is a class of persons, places, objects, events or concepts about which we need to capture and store data. An entity instance is a single occurrence of an entity. The notation is given below:



Student is the name of entity.

   b) **Attribute:** An attribute is a descriptive property or characteristic of an entity. Synonyms include element, property and field. A compound attribute is one that actually consists of other attributes. It is also known as a composite attribute. An attribute "Address" is the example of compound attribute as shown in the following illustration.
   c) **Relationships:** A relationship is a natural business association that exists between one or more entities. The relationship may represent an event that links the entities. The following are some important terms related to ER diagrams:
      ➢ **Cardinality** defines the minimum and maximum number of occurrences of one entity that may be related to a single occurrence of the other entity. Because all relationships are bidirectional, cardinality must be defined in both directions for every relationship. **Figure 7** depicts various types of cardinality.
      ➢ **Degree:** The degree of a relationship is the number of entities that participate in the relationship.

- **Recursive relationship:** A relationship that exists between different instances of the same entity is called recursive relationship. **Figure 6** depicts recursive relationship between the instances of the Course entity.

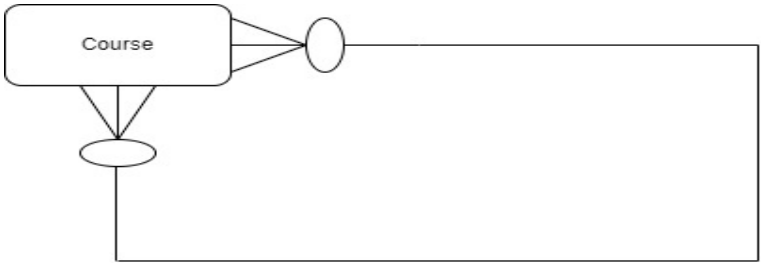


Figure 6: Example of Recursive relationship

Cardinality Interpretation	Minimum Instances	Maximum Instances	Graphic Notation
Exactly one (one and only one)	1	1	OR
Zero or one	0	1	
One or more	1	Many (>1)	
Zero, one or more	0	Many (>1)	
More than one	1	>1	

Figure 7: Different types of cardinality

3) **Decision Tables:** It is very useful for specifying complex policies and decision-making rules. The following are various components of a Decision table:
   a) **Condition stubs:** This portion of table describes the conditions or factors that will affect the decision or policy making of the organization.
   b) **Action stubs:** This portion describes the possible policy actions or decisions in the form of statements.
   c) **Rule:** Rules describe which actions are to be taken under a specific combination of conditions.

d) Decision tables use a standard format and handle combinations of conditions in a very concise manner. Decision table also provides technique for identifying policy incompleteness and contradictions. **Figure 8** depicts a Decision table.

	Rules

X : Action (Condition is true)
__: Condition is irrelevant for this rule
? : Unknown rule

Figure 8: An example Decision Table

4) **Decision Trees:** It is a diagram that represents conditions and actions sequentially, and

Process Name		1	2	3	4	5	6	7	8
Conditions	_____	X	___	.	.	.	.	.	.
	_____								
	_____								
Actions	_____	___	X	?	.	.	.	.	.
	_____								
	_____								

thus shows which conditions to consider first, and so on. It is also a method of showing the relationship each condition and permissible subsequent actions. The diagram resembles branches on a tree. The root of the tree is the starting point of the decision sequence. The particular branch to be followed depends on the conditions that exist and decision that will be made. Progression from the left to right along a particular branch is the result of making a series of decisions. Following each decision point is the next set of decisions to be considered. The nodes of the tree thus represent conditions and indicate that a determination must be made about which condition exists before the next path can be chosen. **Figure 9** depicts a Decision tree.

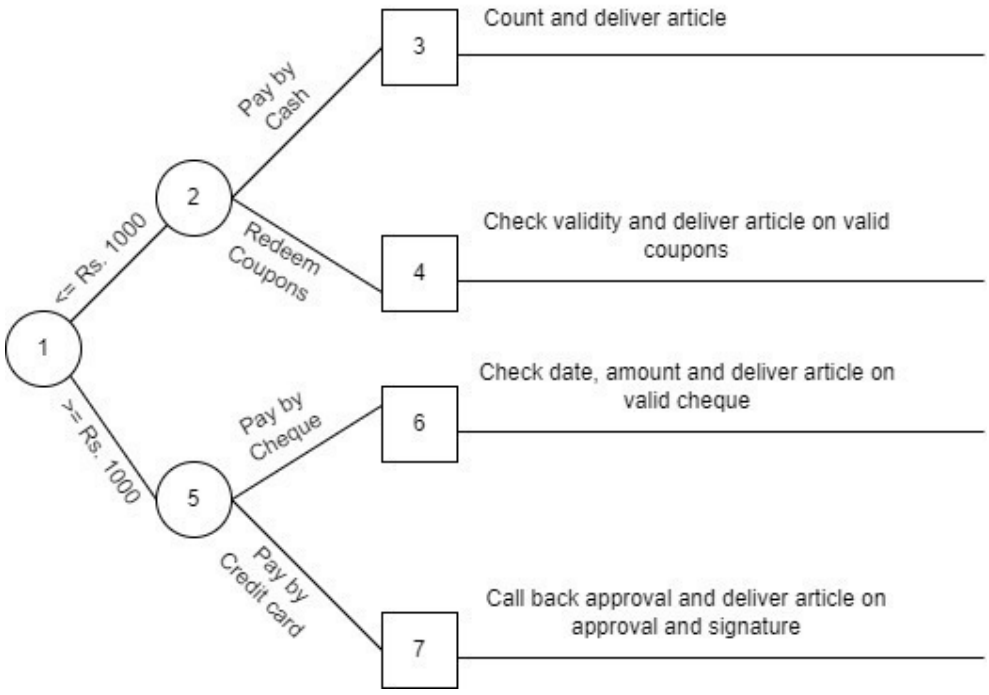


Figure 9 shows an Example Decision tree for a Customer Bill Payment System

5) **Structured English Notation:** It is a tool for describing process. There are three valid constructs in this notation. They are:
   a) A sequence of single declarative statements.
   b) The selection of one or more declarative statements based on a decision, e.g., if-then-else, switch, case.
   c) The repetition of one or more declarative statements, e.g., looping constructs such as do-while, for-do.

The following are the guidelines usage of Structured English notation:

   a) Avoid computer programming language verbs such a move, open or close.
   b) The statement used in the Structured English Notation should always specify the formula to be used.

Structured English notation is based on the principles of structured programming. Process specification logic consists of a combination of sequences of one or more imperative sentences with decision and repetition constructs. Consider the following:

1. An imperative sentence usually consists of an imperative verb followed by the contents of one or more data stores on which the verb operates. For example, add PERSONS-SALARY TO TOTAL SALARY
2. In imperative sentences, verbs such as “process”, “handle” or “operate” should not be used.
3. Verbs should define precise activities such as “add” or compute average etc.
4. Adjectives that have no precise meaning such as “some” or “few” should also not be used in

imperative sentences, because they cannot be used later to develop programs.

5. Boolean and arithmetic operations can be used in imperative statements.

Table 2 lists the arithmetic and Boolean operators.

Arithmetic	Boolean
Multiply (*)	AND
Divide (/)	OR
Add(+)	NOT
Subtract(-)	Greater Than (>)
Exponentiate (**)	Less Than (<)
	Less Than or Equal To (<=)
	Greater Than or Equal To (>=)
	Equals to (=)
	Not Equal To (≠)

6) **Structured English logic:** Structured English uses certain keywords to group imperative sentences and define decision branches and iterations. These keywords are: (BEGIN, END), (REPEAT, UNTIL), (IF, THEN, ELSE), (DO, WHILE), FOR, CASE, etc.

7) **Grouping Imperative Sentences**

1. **Sequence Construct**
   A sequence of imperative statements can be grouped by enclosing them with BEGIN and END keywords.
2. **Decision (Selection)**
   a) A structure, which allows a choice between two groups of imperative sentences. The key words IF, THEN and ELSE are used in this structure. If a condition is 'true', then group 1 sentences are executed. If it is false, then group 2 sentences are executed.
   b) A structure which allows a choice between any number of groups of imperative sentences. The keywords CASE and OF are used in this structure. The value of a variable is first computed. The group of sentences that are selected for execution depends on that value.
3. **Repetition**
   This structure shows two ways of specifying iterations in structured English.
   a) One way is to use the WHILE…DO structure. Here, the condition is tested before a set of sentences is processed. Alternative to WHILE…DO is FOR structure.
   b) REPEAT…UNTIL structure. Here, a group of sentences is executed first then the condition is tested. So, in this structure, the group of sentences is executed at least once.

Table 3 depicts the criteria to be used for deciding the notation among Structured English, Decision Tables and Decision Trees. **Table 4** depicts the criteria to be used for deciding the notation among Decision Tables and Decision Trees.

Table 3: Criteria for deciding the notation to be used

Criteria	Structured English	Decision Tables	Decision Trees
Determining conditions and actions	2	3	1
Transforming conditions and actions into sequence	1	2	1
Checking consistency and completeness	2	1	1

Table 4: Criteria for deciding the notation used between Decision tables and Decision trees

Criteria	Decision Tables	Decision Trees
Portraying complex logic	Best	Worst
Portraying simple problems	Worst	Best
Making decisions	Worst	Best
More compact	Best	Worst
Easier to manipulate	Best	Worst

8) **Data Dictionary:** A Data Dictionary consists of data about data. The major elements of data dictionary are data flows, data stores and processes. The data dictionary stores details and descriptions of these elements. It does not consist of actual data in the database. But, DBMS cannot access data in database without accessing data dictionary.

   If analysts want to know the other names by which a data item is referenced in the system or where it is used in the system, they should be able to find the answers in properly developed data dictionary. Data dictionaries are hidden from users so that data in it is not tampered. Analysts use data dictionaries for the following reasons:

   1. To manage the detail in large systems.
   2. To communicate a common meaning for all system elements.
   3. To document the features of the system.
   4. To facilitate analysis of the details in order to evaluate characteristics and determine changes that should be made to the system.
   5. To locate errors and omissions in the system.

The dictionary contains two types of descriptions for the data flowing through the system: Data elements and Data structures. Data elements are grouped together to make up a data structure. Data elements are recorded in data dictionary at the fundamental data level. Each item is identified by a data name, description, alias and length and has specific values that are permissible for it in the system. A data structure is a set of data items that are related to one another and then collectively describe a component in the system. Data is arranged in accordance with one of the relationships namely sequence, selection, iteration and optional relationship.

4.7 INPUT/OUTPUT DESIGN

Input Design

In an information system, input is the raw data that is processed to produce output. During the input design, the developers must consider the input devices such as PC, MICR, OMR, etc. Therefore, the quality of system input determines the quality of system output. Well designed input forms and screens have following properties:

- It should serve specific purpose effectively such as storing, recording, and retrieving the information.
- It ensures proper completion with accuracy.
- It should be easy to fill and straightforward.
- It should focus on user's attention, consistency, and simplicity.
- All these objectives are obtained using the knowledge of basic design principles regarding:
  - What are the inputs needed for the system?
  - How end users respond to different elements of forms and screens.

Objectives for Input Design

- To design data entry and input procedures
- To reduce input volume
- To design source documents for data capture or devise other data capture methods
- To design input data records, data entry screens, user interface screens, etc.
- To use validation checks and develop effective input controls.

Data Input Methods

It is important to design appropriate data input methods to prevent errors while entering data. These methods depend on whether the data is entered by customers in forms manually and later entered by data entry operators, or data is directly entered by users on the PCs.

A system should prevent user from making mistakes by:

- Clear form design by leaving enough space for writing legibly.
- Clear instructions to fill form.
- Clear form design.
- Reducing key strokes.
- Immediate error feedback.

Some of the popular data input methods are:

- Batch input method (Offline data input method)
- Online data input method
- Computer readable forms
- Interactive data input

Input Integrity Controls: Input integrity controls include a number of methods to eliminate common input errors by end-users. They also include checks on the value of individual fields;

both for format and the completeness of all inputs. Audit trails for data entry and other system operations are created using transaction logs which gives a record of all changes introduced in the database to provide security and means of recovery in case of any failure.

Output Design

The design of output is the most important task of any system. During output design, developers identify the type of outputs needed, and consider the necessary output controls and prototype report layouts.

Objectives of Output Design

- To develop output design that serves the intended purpose and eliminates the production of unwanted output.
- To develop the output design that meets the end users requirements.
- To deliver the appropriate quantity of output.
- To form the output in appropriate format and direct it to the right person.
- To make the output available on time for making good decisions.

Types of Output:

1. External Outputs

Manufacturers create and design external outputs for printers. External outputs enable the system to leave the trigger actions on the part of their recipients or confirm actions to their recipients. Some of the external outputs are designed as turnaround outputs, which are implemented as a form and re-enter the system as an input.

2. Internal outputs

Internal outputs are present inside the system, and used by end-users and managers. They support the management in decision making and reporting. There are three types of reports produced by management information:

- **Detailed Reports** − they contain present information which has almost no filtering or restriction generated to assist management planning and control.
- **Summary Reports** − they contain trends and potential problems which are categorized and summarized that are generated for managers who do not want details.
- **Exception Reports** − they contain exceptions, filtered data to some condition or standard before presenting it to the manager, as information.

Output Integrity Controls: Output integrity controls include routing codes to identify the receiving system, and verification messages to confirm successful receipt of messages that are handled by network protocol. Printed or screen-format reports should include a date/time for report printing and the data. Multipage reports contain report title or description, and pagination. Pre-printed forms usually include a version number and effective date.

4.8 FORMS/ REPORT DESIGN

Forms Design

Like a form on paper that is used to fill out information with a pen or pencil, a Form in computer terminology identifies the data we want to collect. It also allows us to enter data into the database, display it for review and also print it for distribution. However, an electronic form has several important advantages over standard paper forms. These have the advantage of using a computer database and are more versatile and powerful than paper forms. Examples of forms are Business forms, Electronic spread sheet, ATM transaction layout, etc.

Figure 10 shows a simple form that is used to collect employee details.

Importance of Forms:

- A form provides an easy way to view data.
- Using forms, data can be entered easily. This saves time and prevents typographical errors.
- Forms present data in an attractive format with special fonts and other graphical effects such as colour and shading.
- Forms offer the most convenient layout for entering, changing and viewing records present in the database.
- An entry field in a form can present a list of valid values from which users can pick to fill out the field easily.

Reports Design

- Analyzing and presenting data are just as important as entering and sorting these out. Computer systems use reporting and query applications to retrieve the data that are available in the database and present it in a way that provides useful information, drives decision-making and supports business projects. A report presents data as meaningful information, which can be used and distributed.
- A report is the information that is organized and formatted to fit the required specification. It is a passive document that contains only predefined data and is used solely for viewing and reading. Reports can be printed on paper, or these may be

transferred to a computer file, a visual display screen, etc. Reports are the most visible component of a working information system and hence they often form the basis for the users and management's final assessment of the systems value.

- Examples of reports are: invoices, weekly sales summaries, mailing labels, pie chart, etc.

EMPLOYEE RESIDENCE PHONE LIST				
Sl.No.	**Last Name**	**First Name**	**Designation**	**Phone Number**
1	Verma	Ajay	Regional Manager	6522081
2	Gupta	Vinay	Branch Manager	6478017
3	Michael	Nancy	H. R. Manager	6152430
4	Singh	Amar	Sales Executive	5769081

Figure 11 shows a simple report that displays the residence telephone numbers of all the employees in the organization.

Importance of Reports:

- We can organize and present data in groups.
- We can calculate running totals, group totals, grand totals, percentage of totals, etc.
- Within the body of Reports, we can include sub-forms, sub-reports and graphs.
- We can present data in an attractive format with pictures, special fonts and lines.
- We can create a design for a report and save it so that we can use it over and over again.

Differences between Forms and Reports

1. Forms can be used for both input and output. Reports, on the other hand, are used for output, i.e., to convey information on a collection of items.
2. Typically, forms contain data from only one record, or are at least based on one record such as data about one student, one customer, etc. A report, on the other hand is only for reading and viewing. So, it often contains data about multiple unrelated records in a computer file or database.
3. Although we can also print forms and datasheets, reports give more control over how data are displayed and show greater flexibility in presenting summary information.

PROCESS OF DESIGNING FORMS AND REPORTS

Good quality business processes deliver the right information to the right people in the right format and at the right time. The design of forms and reports concentrates on this goal. Designing of forms and reports is a user-focused activity that typically follows a prototyping approach. Before designing a form or a report, we should have a clear idea so as to what is the aim of the form or report and what information is to be collected from the user. There are some useful questions related to the creation of all forms and reports, such as "who, what,

when, where and how" which must be answered in order to design effective forms and reports.

WHO	Understanding who the actual users are, their skills and abilities, their education level, business background, etc., will greatly enhance the ability to create effective design.
WHAT	We need to have a clear understanding of what is the purpose of the form or report and what task will the users be performing and what information is required so as to successfully complete the given task.
WHEN	Knowing when exactly the form or report is needed and used will help to set up time limits so that the form or report can be made available to the users within that time frame.
WHERE	Where will the users be using this form or report (i.e., will the users have access to on-line systems or will they be using them in the field)?
HOW	How many people will be using this form or report, i.e., if the form or report is to be used by a single person, then it will be simple in design but if a large number of people are going to use it, then the design will have to go through a more extensive requirements collection and usability assessment process.

After having answered all the above questions, we would have collected all the initial requirements. The next step is to refine this information into an initial prototype. Structuring and refining the requirements are completed without interacting with the end users, although we may need to occasionally contact users in order to clarify some issues that might have been overlooked during analysis.

Once the initial prototype is ready, we should ask the users to review and evaluate the prototype. After the review, the design may be accepted by the users. Or at times the users may ask for certain changes to be made. In case changes are to be made then the construction-evaluation-refinement cycle will have to be repeated until the design is accepted. The next step in the Design process is to Design, Validate and Test the outputs using some combination of the following tools:

a)	Layout tools	(Ex.: Hand sketches, printer/display layout charts or CASE)
b)	Prototyping tools	(Ex.: Spreadsheet, PC, DBMS, 4GL)
c)	Code generating tools	(Ex.: Report writer)

The initial prototype may be constructed in numerous environments. For example, a CASE tool or the standard development tools that are used within the organization be used. Usually, initial prototype are mock screens that can be produced using word processor, computer graphics design package, or electronic spreadsheet. Mock screens are not the working modules or systems.

Tools for designing forms and reports are rapidly evolving and now a days Online graphical tools for designing forms and reports are very much in use in most professional development organizations.

CRITERIA FOR FORM DESIGN

Forms should be well conceived and attractive in design. We can achieve this goal if we design a form that satisfies the following criteria:

1. ***Organization*:** The different parts of a form must be arranged in a proper order with visual separation between the parts. Balancing of different information on the form should be done according to the sequence of entry, frequency of use, function and significance of that particular data. The first data available, the most important data and the data that is going to be used most frequently should always be placed in the beginning of the form. If there are groups of data of the like information, they should be placed together just as Name+Address+Phone Number. Grouping of information will help the user to understand which section of the form they are completing.
2. ***Consistency*:** Forms designed should be internally consistent. They must also be consistent with related forms and with other forms in the organization. If the forms are consistent, then it will be easy for the users to learn how to fill them. Consistent forms reduce errors and data capture costs.
3. ***Completeness*:** The form should gather all the necessary data at the source so that there is no need to transcribe data to other forms. This reduces the major source of errors.
4. ***Flexible Entry*:** It should be possible to enter data by hand or with a typewriter. In most cases, both kinds of entries occur.
5. ***Economy*:** The total cost of design, printing, data entry, etc., must be minimized. Most of the times, it is required to increase one cost to reduce another. Usually, handling costs are much more than the cost of designing and printing. Having spent more resources on design and printing often reduces the cost of data capture and keying.

CRITERIA FOR REPORT DESIGN

Reports convey information from one of more computer files to the user. They perform this task satisfactorily only when they present information to the user accurately and in small portions. Several criteria that should be considered in order to produce good reports are given below:

1. **Relevance:** Only the information that is relevant to the purpose of the report should be present in the report. This is a selection process, i.e., all the relevant information should be included and all the irrelevant information or data should be excluded. Only required information should be printed or displayed. In on-line reports, we should use information hiding and provide methods to expand and contract levels of information details.
2. **Accuracy:** The data that appears on the report should be accurately recorded, accurately transmitted and accurately transformed into summary data. Accuracy is very important because if the data are inaccurate, then the main purpose of the report which is to provide accurate information to the user will not be accomplished. Incomplete data are also inaccurate.

3. **Clarity:** The information that is present on the report should be clear and understandable. The information present should be balanced on the report, the display should not be too crowded and not too spread out. Sufficient margins and spacing throughout the output will enhance readability. Desired information must be easy to locate. Comparisons, ratios, percentages, exception flags and graphs should be used where necessary.
4. **Timeliness:** Reports must be prepared and ready for use in time. Most reports provide information, which is used to make decisions. Hence, this information must reach the recipients while the information is pertinent to transactions or decisions. Information is of very little use if it arrives after the decisions are made.
5. **Cost:** Every report has two costs. First is the cost of preparation, which consists of analysis, design, computation and distribution. Second is the cost of reading the report and locating germane parts of it. Often the cost of reading the report is forgotten during the calculation of costs. The reading cost can be significantly reduced only if the appropriate information is presented clearly on the report. The total cost should always be less than the expected benefits. Only then the report should be prepared.

Check Your Progress 2

1. Define Form and Report. Also, differentiate between them.
2. List the advantages of using Forms and Reports.
3. Describe the process of designing Forms and Reports.
4. State True or False for the following:
   a) A form is a type of layout that typically lists and summarizes data from several unrelated data base records.
   b) A report is a type of output that typically lists data associated with one data base record.
   c) Within reports, we can include sub-forms and sub-reports.

4.9 DESIGN OF PHYSICAL FILES

Files are collection of logically connected records. In RDBMS, files are called tables. However, the ways the files are stored in memory depend on the operating system. Many operating systems allow files to be split into pieces but the same is transparent to the user.

Types of Files

1. A **Master file** is a permanent file of all the data needed by a business. Some of the fields may be regularly updated from transactions. For example, a file consisting of information about customers.
2. A **Transaction file** is a temporary file of all the transactions (items bought, sold, deleted etc.) that have taken place in a given period. It stores data related to day to day business transactions. For example, data related to daily sales activity. The contents of these files are used to update the master file. Daily sales are used to update balance in the customer file.
3. An **Archive file** is a file of data in permanent storage (usually, the data is stored for legal reasons or to perform trend analysis). Archive files will contain all information about the

past dealings of the business and would normally be stored on a different site to facilitate recovery in case of a disaster such as fire.

4. An **Audit file** is a file that does not store business data but data related to transaction log. For example, data and time of access, modification etc. of data , values of fields before and after modification etc.
5. A **Work file** is file temporarily created to hold intermediate result of the data processing. For example, a sorted files of list of customers.

File Organization

File organization is the physical organization of records on the disk. There are different types of file organizations depending on the organization of records in the disk and other secondary storage. Before deciding on a specific file organization, we should ensure that its application leads to the following:

- Fast retrieval of records
- Reduce disk access time
- Efficient use of disk spaces

Serial file organization

A serial file is created by placing the record as it is created. It leaves no gap between the records that are stored on the disk. The utilization of space called packing density approaches 100 percent in this case. Examples of serial files are print file, dump file, log files, and transaction files. These files are created once and are not used for addition or deletion or any kind of record searching operation.

Sequential file organization

In this organization, the records are physically ordered by primary key. To locate a particular record, the program starts searching from the beginning of the file till the matching primary key is found. Alphabetic list of customers is a common example of sequential file organization. Deletion of record may cause wastage of space and adding a new record requires rewriting of the file. This type of file organization is suitable for master files and is not used where fast response time is required.

Indexed sequential file organization

In this organization, records are not physically ordered. Index is created to facilitate searching of records. Index Records give physical location of each data record. Indexes are separate files with a link to the main file. This type of file organization is used where faster response time is required. Figure 9.8 depicts Indexed sequential file organization.

Product_no	Name	Type	Price
234	Processor	C	5660
235	Harddisk	D	3400
236	Memory	B	1400

Type	Pointer
B	5660
D	3400
C	1400

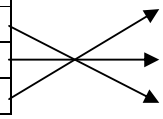


Figure 12: Indexed Sequential File Organization

Hashed file organization

In this organization, records are physically ordered according to hashing algorithm. The address where each record is stored is determined using hashing algorithms. **Figure 13** depicts an example of a Hashed file organization. The following is a typical hashing algorithm:

1. Uses a field in record called the hash field (generally the key filed).
2. Divides by prime number known as hash function.
3. Produces record address called the hash address.

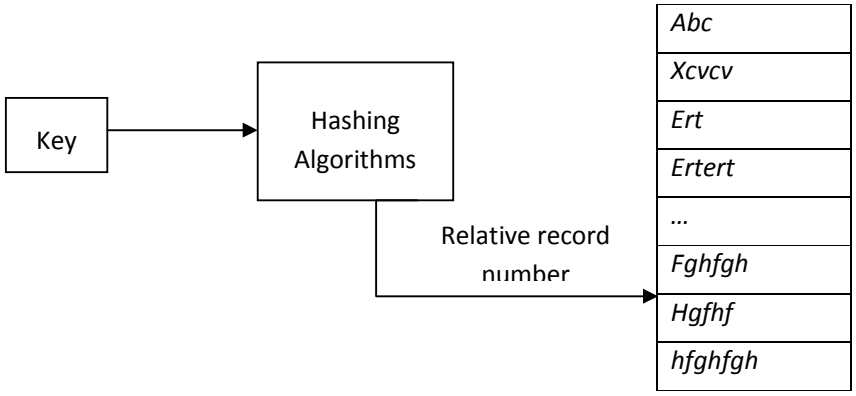


Figure 13: Hashed File Organization

Check Your Progress 3

1. The Records are physically ordered by primary Key in _____ file organization.
2. _____ type of file organization is suitable for master files.

4.10 INTRODUCTION TO DATABASE DESIGN

Over a period of time, massive advancements have taken place in the field of databases. Storage of data and retrieval is an integral part of any information system.

Flat Files vs. Database

Traditionally, data are being kept in flat files. Consider an example of flat file which stores pin code of customer's address. Any change in the size of the field will enforce changes in the entire program which uses the data related to customers' address. If the file is being used by more than one application, the problem can become even worse.

Steps in Database Design

Analysis is the process of creating a conceptual data model independent of the target database technology. The typical result is an ER model. Design is the process of creating a logical data model. This step is dependent on the target technology (relational, hierarchical, or network), but not on the specific database implementation (such as Oracle, MS SQL, DB2 for OS/390 or DB2 Universal Database). Implementation is the process of creating a physical model or schema for one specific database system, such as Oracle, MS SQL, DB2. The result is an optimized physical design.

E-R Model to Database Design

The process of database design process involves the task of converting logical model to working physical model. The logical model can be arrived at by the application of normal forms. Normal forms are nothing but a set of rules applied sequentially starting from the basic table to the table resultant due to the application of a normal form. As of today, there are a total of five normal forms. The first normal form is applied to the basic table. The resultant table is given as input to the second normal form and so on. It is not mandatory to continue this process until the fifth normal form. Usually, the process is continued until third normal form. Fourth and fifth normal forms are applied only when there are multivalue dependencies and join dependencies respectively. We are not introducing you to the normalization theory in this course. It will be dealt in the course related to databases in detail. Physical database design starts from a given relational model. That is, from the definition of a set of tables and their respective columns. The objective of physical database design is to fulfill the performance requirements of a set of applications by optimizing the use of the DBMS. Key areas include: optimizing the index configuration, data placement and storage allocation.

Entities with one-to-one relationship should be merged into a single entity. A table models each entity with a primary key and non-key attributes, some of whom may be foreign key(s). One-to-many relationships are modeled by a foreign key attribute in the table representing the entity on the "many" side of the relationship. Many-to-one relationship between two entities is modeled by a third table that has foreign keys that refer to the entities. These foreign keys should be included in the primary key of the relationship table, if appropriate. Many commercially available tools can automate the process of converting an E-R model to a database schema. **Figure 14** depicts various steps involved in the design of databases.

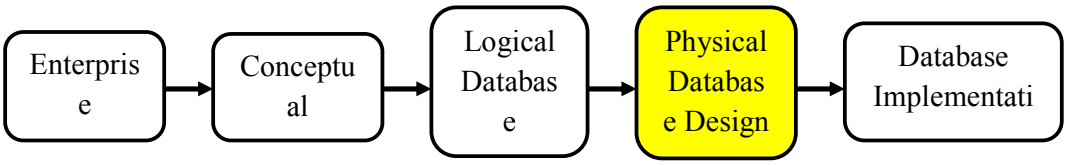


Figure 14: Steps in Database Design

Inputs to Physical Database Design

1. Logical structure of Database (Normalized relations).
2. Definition of attributes – data type, integrity control, error handling.
3. Choice of RDBMS
   a) ***Hierarchical*:** Hierarchical database structure is a kind of database management system that links records in tree data structure such that each record has only one owner. For example an order is owned by only one customer. It resembles a tree structure.
   b) ***Network*:** Network database structure is more flexible structure than Hierarchical model as well as relational model, but not preferred due to the high processing time. A neural network is an example of network database.
   c) ***Relational*:** Relational database structure is most commonly used database model. The data is modeled as a mathematical relation. Reference key joins different tables together. Indexes provide rapid access to specific record in the database. For example, DB2, MySQL, Oracle are some RDBMS. Object relational (Oracle 8i/9i/10i)
   d) ***Object Oriented:*** Object Oriented Database management system is based on object oriented paradigm. In object oriented database, data is stored as objects and can be interpreted only by using the methods specified by its class.
4. Estimation of database size growth rate and frequency of usage.
5. Requirements for backup, recovery, response time and retention time.

Guidelines for Database Design

1. Ensure that the data stored in the files (database tables) are atomic. Data stored in the atomic form can be combined later to generate data in specific form.
2. Every table must have a primary key which identifies each record in the table distinctly. Descriptive and meaningful name is to be used while naming a field in the table (For example, use product_id instead of ID).
3. Use single column primary key whenever possible. As most of the join operations are made on primary key and composite primary keys make the operation slower.
4. Use numeric key whenever possible.
5. Use primary key name as foreign key for better readability.
6. Avoid allowing null values to go into the columns that have discrete range of possible values and
7. Avoid multiple tables with similar structure when one table is sufficient.

Figure 15 depicts various records and fields in a file which consist of details of various employees in a hospital.

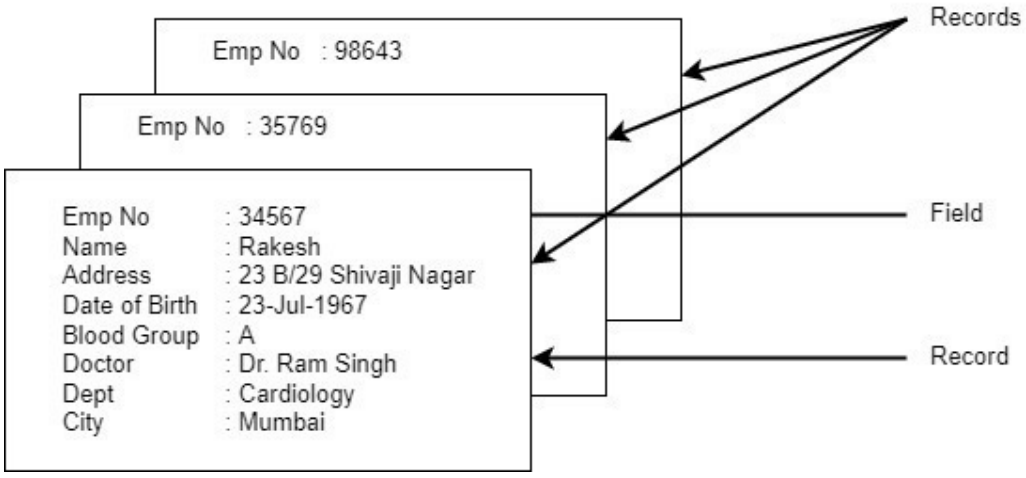


Figure 15: Records and Fields

DESIGN OF DATABASE FIELDS

Attributes in E-R model are known as fields in physical data model. A field is the smallest unit of data that is stored and manipulated. Fields are used in conventional files as well as in databases. It is the implementation of attributes and can be termed as smallest unit of meaningful data.

Types of Fields: The following are various types of fields in databases:

Primary key: Any conventional file system or database stores two kinds of field namely *descriptive fields* and *primary key*. Descriptive fields comprise the customer names, inventory numbers, item descriptions, and so on, which are used by the application. Keys refer to the *primary* and *foreign* keys that are used to find database records and relate them to one another. Keys are fundamental to the concept of relational databases because they enable tables in the database to be related with each other.

A table must have a *primary key* i.e. an attribute or combination of attributes that are guaranteed to be unique and not null. It is sometimes helpful to introduce a surrogate field to act as a key. This could be a table attribute, which has no business meaning, but simply added to serve as a unique identifier for each record in the table. This is sometimes referred to as plumbing. The requirements for a primary key are very hard. It must conform to the following rules:

- They should exist.
- Be unique in the table.
- The values must not change or become null during the life of each entity instance.
- It must have a not-null value for each instance of the entity.

Surrogate keys are often required because sometimes, real business data does not fulfill the requirement of a primary key. Furthermore, the surrogate key is typically a single field (not a

composite key), which simplifies the database schema, particularly when the key is used in other tables as a foreign key.
Most of modern RDBMS are tuned in for queries on integers, so it is advisable to use this data type as a primary key. Many RDBMS provide a special serial number or sequence number of integer type, which generate a sequence of unique integers as a row is inserted into the table. Declaring a column to be of this type guarantees that a unique key is generated for each inserted row.

Secondary key: Also known as Alternate key. This is a field or collection of fields in the table which can be used as primary key in addition to the already existing primary key.
Foreign keys are table attributes, the values of which are the same as those of primary keys of another table. It is often desirable to label foreign key columns explicitly. For instance, by adopting a naming convention. Existence of foreign key enforces the referential integrity constrains (discussed later in this Unit). A referential integrity constraint (references) should be declared as part of the CREATE statement in a DBMS while creating the table.

Descriptive fields: Attributes that are not used as key but store business data.

RULES FOR NAMING TABLES AND FIELDS

1. ***Names for all database elements should be:***
   - Unique
   - Meaningful
   - Short
2. ***Restrictions for naming tables:***
   - Use no acronyms or abbreviations. Should be descriptive to convey meaning.
   - Should not imply more than one subject
3. ***Restriction for naming fields:***
   - No acronyms
   - Use abbreviations only if clear and meaningful
   - Should not imply more than one subject
   - Should be singular.

While designing database fields, it is required to set the properties of the fields.
Name: A name is used to refer the attribute in the DBMS that uniquely labels the field. The name of the attribute in the logical data model and the name of the field in the physical data model must be same. For example, student name in a student table.

Data type: Type of data the field is expected to store. This could be numeric, alphanumeric etc. The data type, supported by various RDBMS varies to a great extent. For example, student_name CHAR (25), indicates that the name of the student is of character data type, 25 indicates the maximum size of the data that can be stored in the field. The data type selected should ensure the following:

- It involves minimum usage of memory and represents all possible values

- Supports all types of data manipulation that is expected from the business transaction.

Size: It indicates the size of the database fields. Many RDBMS support sizes that are variable. For example, VARCHAR data type in Oracle.

Null or not null: specifies whether the field will accept null value. Not null constrains applied in DBMS ensure that null values are not entered to the respective fields. A null value is a special value distinct from 0 or blank. A null value indicates that the value is either missing or unassigned yet. We may specify that customer_name in a customer table to be not null. When a field is declared a primary key DBMS automatically ensures that the field in not null.

Domain: It indicates the range of values that are accepted by the fields. For example: Basic_Pay in an employee table can assume any value between the lowest basic_pay and highest basic_pay existing in the company. In such cases, the value of the field can be restricted to the one between the highest and lowest value to avoid entry of non-existing basic_pay.

Default value: It refers to the value that is stored by default in the field. For example, ship_date in an invoice is most of the time same as invoice_date (current date). When a default value is assigned to a field, it reduces a lot of data entry time and reduces the chances of error.

Referential integrity: It refers to a set of rules that avoid data inconsistency and quality problems. Referential integrity ensures that a foreign key value cannot be entered unless it matches a primary key value in another table. RDBMS automatically enforces the referential integrity once the database designer identifies and implements primary and foreign key relationship.

- It prevents orphaned records. I.e. when a row containing a foreign key is created, the referential integrity constrains enforced vide the RDBMS ensure that the same value also exists as a primary key in the related table.
- When a row is deleted, it should be ensured that no foreign key in related tables is the same value as primary key of the deleted row.

Figure 16 depicts primary and foreign keys for two tables namely customer and order.

Customer_id	Customer_name	Customer_city	Customer_Phone
5466	John	New Delhi	2345678
5678	David	Mumbai	2567890

Table: Customer

Customer_id	Customer_name	Customer_city	Customer_Phone
5466	John	New Delhi	2345678
5678	David	Mumbai	2567890

Table: Order

Figure 16: Primary key and foreign key relationship

Customer_id is the primary key in **customer** table and is a foreign key in **order** table. This referential integrity constraint ensures that the value customer_id in **order** table must exist in the **customer** table. The primary key is shown in bold and the foreign key in italics.

DESIGN OF DATABASE

Database design is similar to the pillars of a building. Any negligence, errors in Database design may lead to degraded performance of the software. In some cases such as real time applications, it may also lead to disasters. The following are various steps in Database design:

1) **SELECTION OF DATABASE ARCHITECTURE**

Selecting database architecture is one of the most challenging parts of database design for any information system**.** Before deciding on the target DBMS where the database is to be implemented, few considerations are required.

a) **Hierarchical database** structure is a kind of database management system that links records in tree data structure such that each record has only one owner. For example an order is owned by only one customer. It resembles a tree structure.
b) **Network database** structure is more flexible structure than Hierarchical model as well as relational model, but not preferred due to the high processing time. A neural network is an example of network database.
c) **Relational database** structure is most commonly used database model. The data is modeled as a mathematical relation. Reference key joins different tables together. Indexes provide rapid access to specific record in the database. For example, DB2, MySQL, Oracle are some RDBMS.
d) **Object Oriented Database** management system is based on object oriented paradigm. In object oriented database, data is stored as objects and can be interpreted only by using the methods specified by its class.

2) **DESIGNING OF DATABASE SCHEMA**

A blue print of the database is a physical model. A schema defines the database in terms of tables, keys, indexes and integrity rules. A **relational schema** consists of a relation, name of the attributes in the relations and restrictions on the relations called integrity constraints. A **database schema** is a set of relation schemas. Changes to a schema or database schema are expensive. So, careful thought must be given to design of a database schema. The following are some guidelines for the design of a database schema.

a. Each entity should be implemented as a database table.
b. Each attribute should be implemented as a field.
c. Each table must have a primary key and an index based on the key.
d. Each table may have zero or more secondary keys.

e. Appropriate foreign keys.

The following is an example of a database schema:
employee (emp_name, birth_date, social_security_no, dept_name),
department (dept_name, function).

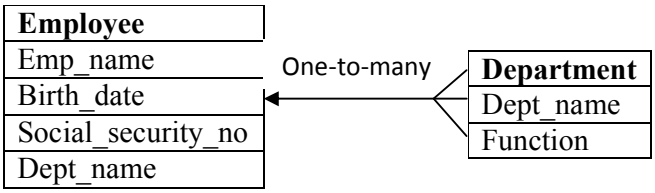


Figure 17: A Database Schema

A database schema defines a database in terms of tables, keys, indexes and constraints.
The following are various objects of a schema:

- Fields
- Records
- Tables
- Database

1. **SELECTING INDEXES**

During the process of file and database design one must choose index based on a single field (usually the primary key) or multiple fields. While selecting index, one must keep in mind the performance issues vis-à-vis the issue of inserting and deleting records. While indexes can be used generously on tables primarily used for query purpose with rare necessity to update records, they should be used judiciously in tables that support transaction processing which involve large insertion, updation and deletion operations. The amount of time needed to maintain the indexes in database tables increases with the number of rows stored. When an index is created on a table, a separate storage area is allocated to store the index structure. A database table can have one or more indexes associated with it.
For example, consider an **employee** record where an index created on the EMP_ID field of the **employee** table. The index table contains a sorted list of the employee ID values. Indexes may significantly improve the performance of SQL queries. It may not be noticed with small tables but it can be quite significant for larger tables. However, there are disadvantages to having too many indexes on table. Indexes can slow down the speed of some inserts, updates, and deletes when the DBMS has to maintain the indexes as well as the database tables. Also, indexes take additional disk space as they are stored separately.

2. **ESTIMATING CAPACITY OF THE DATABASE**

Database administrator needs to calculate the amount of disk space required for the database.

1. In a given table, calculate the record size by adding the sizes of various fields in it.
2. Also, the number of records that will be present in each table at a particular period of time should be forecast.

Table size = record size * number of records in the table.

Database size is sum of sizes of all tables in that database. As rule of thumb, add a factor of 50% for indexes and other overheads to get the expected database size. While designing a database, future growth of database should also be kept in mind. Most of the business databases have a liner growth trend.

Check Your Progress 4

1. The process of Database Design involves the task of converting logical model to _____.
2. _____ types of relationship should be merged to a single entity.
3. Commercially available tools can automate the process of converting a _____to a database schema.
4. Attributes in E-R model are known as _____ in physical model of data.
5. _____ are often required because real business data sometimes does not fulfill the requirement of existence of a primary key.
6. A set of rules that avoid data inconsistency and quality problems is called _____.
7. _____ is a blue print of the database.
8. A neural network is an example of _____.
9. Indexes take additional disk space? Yes/No.

4.11 Summary

This unit focussed on the requirements of a good design. The two design process, namely, Top-down Design and Bottom-up Design have been explained. Analysis and design phase are discussed mainly in the stages of system design. The various design methodologies such as Data Flow Diagrams, E-R diagrams, Decision Tables, Decision Trees and Structured English notation are explained in this unit.

Design is the phase that precedes coding. All the components of system are designed logically. Then physical aspects of the system are designed. Form design, report design, database design and program design etc. are designed with the help of GUI controls, process modeling tools, data modeling tools. These tools reduce the complexity of the design process. A Data Dictionary is data about data. These elements centre on data and the way they are structured to meet user requirements and organization's needs.

Here, we have seen that forms and reports are instruments of communication between people and computers. A well designed form should satisfy the criteria like Organization, Consistency, Completeness, Flexible entry, Economy and a well design report should satisfy the criteria like relevance, accuracy, clarity, timeliness, cost, etc.

Traditionally, file is being used for keeping data. Use of DBMS has been the standard to store data for today's information systems due to their various advantages. Any physical database design involves steps like choice of target DBMS on which the database is going to be implemented. Relational database is mostly being used unless the application has specific

requirements. The physical database design process can be considered as a mapping from logical model to physical working database, which involves design of fields, design of records and finally design of the database. While transforming the logical model to physical model, many implementation issues related to the information system and target DBMS are to be addressed. Database volume estimation is an important part of database design. The present size and future growth of database is to be estimated before implementing the database.

4.12 Solutions/Answers

Check Your Progress 1

1. Abstraction
2. Top-down Design
3. Bottom-up Design
4. Structure Charts
5. Coordinating module , Subordinate modules
6. Lower level

Check Your Progress 2

1. In computer terminology, a Form identifies the data we want to collect. A report is the information that is organized and formatted to fit the required specification. The difference between Form and Report is that a form is used to collect or present information and reports are used to convey information on a collection of items.
2. The advantages of using the forms are given below:
    - Forms provide an easy way to view data.
    - Using forms, data can be entered efficiently.
    - We can present data in attractive format using forms.
    - Forms offer convenient layout for entering, changing and viewing records present in the database.

    The advantages of reports are:
    - Reports organize and present data in groups.
    - With reports, we can calculate running totals, group totals, grand totals etc.
    - We can present data in attractive format.
    - Within reports we can include sub-forms, sub-reports and graphs.
3. While designing Forms and Reports, the following steps have to be taken:
    a) The first is to collect all relevant information regarding the form or Report by asking questions like who, what, when, where and how.
    b) Refine the above information into an initial prototype.
    c) Review and evaluate the prototype.
    d) Design, validate and test the outputs using some combination of prototyping and code generation tools.
4. State True or false
    a) False
    b) False
    c) True

Check Your Progress 3

1. False
2. False

Check Your Progress 4

1. Working physical model
2. One-to-one
3. E-R Model
4. Field
5. Surrogate keys
6. Referential integrity rules
7. Database Schema
8. Network database
9. Yes

4.13 Further Readings

1. Joey George, J. Hoffer and Joseph Valacich; *Modern Systems Analysis and Design*; Pearson Education; Third Edition; 2001.
2. Alan Dennis, Barbara Haley Wixom; *Systems Analysis and Design;* John Wiley & Sons; 2002.
3. John W. Satzinger, Stephen D. Burd, Robert B. Jackson; Systems Analysis and Design in a changing world; Course Technology Inc.; Third Edition;2004.
4. Elias M. Awad; Systems Analysis and Design; Galgotia Publications; Second Edition; 1994.

4.14 Probable Questions

1. Write briefly about the top-down and bottom-up design approach.
2. List the tools and techniques used for describing the system design of the system.
3. Explain the Data Flow Diagram with an example.
4. Describe briefly about the design methodologies used in the system.
5. Specify the criteria used to identify a good form design.
6. Specify the criteria used to identify a good report design.
7. Differentiate between logical design and physical design.
8. Write the different rules for designing the user interface design.
9. Define the term Input design and output design.
10. Describe the types of outputs.
11. What is the difference between forms and reports?
12. Write the different types of files.
13. Explain the file organization.
14. Define the terms: primary key, foreign key, surrogate Keys, composite Keys, field, record, table, Database Schema.
15. Explain the various steps in database design.
16. Consider the following tables :

Employee (**e_number**, Title, Last_name, First_name, Supervisor_emp_no)
Department (**number**, Name, Manager)
Location (**dept_no**, Location)
Dependent (**Emp_number**, name, relationship)
Draw an Entity Relationship diagram for Employee Database.

BLOCK IV : Unit-V
System Implementation

Unit Structure:

5.1 Introduction

Implementation of System is the sixth step of the SDLC life cycle. After the design phase is over, computer programs are actually written in accordance with the design specification. Implementation of the system not only includes coding the software, but also includes testing the complete system, called system testing, and also creating the required hardware and networking environment for the smooth operation of the system. It also includes imparting training to the end users .Quality assurance which ensures that the system meets the requirements and the specifications for its intended use and performance is also guaranteed. A thorough conversion plan is prepared to replace the existing system with the new built system. A well designed conversion plan guarantees a smooth transition from the existing to the new system. Apart from this, database is designed and created and installed either by using the existing data which is retrieved from the old system or by manually inserting data. Proper training of the user is important because the key to success for any system is dependent on the participation of end user as they are the one who will actually use the system. A detailed user manual is prepared, mentioning the procedural things that the user can refer to in order to use the system. Transition to the new system can be carried out either in in just one instance or in phased manner depending on the situation. This phase of the system development life cycle requires active user involvement.

5.2 Objective

The aim of this unit is to put light on some of the activities carried out as part of the system implementation process. After going through this unit, you will be able to understand the following

- Various System tests
- Software quality assurance
- Software and hardware specification
- Security Aspects

5.3 System Testing

No system can ever be perfect. This makes testing an important part of implementation because a team of Software Engineers design and develop the information systems with end users having little or no knowledge about system development. Testing is carried out in order to bridge the gap between the expected outcomes desired by the user to that of the systems analysts and programming team. The design specifications that reflect the user requirements are translated by the programmer into working software. Hence, the success of the

programmer to code the software exactly as per the design specification can be judged by only by testing the software system. In other words, testing is a set of process or activities that checks the intended functionality and correctness of the software with regards to user specification.

The aim of any testing mechanism is to find errors and fix them before the product can be delivered to the customer. A good testing strategy has high possibility of finding an error that remained undiscovered. The main objective of any testing strategy is to discover and correct the errors with minimum resources and. Apart from errors and bugs, systems are tested for security and usability, response time, the volume of transactions the system can handle, amount of stress under which it can operate.

System testing works with the assumption that all its parts are correct and has no error. Even though the individual modules of the system have been tested, yet it cannot be guaranteed that after integration of the various modules, the system will work according to the desired specification. System testing aims at testing the working of the application as a whole.

Testing is carried out in order to bridge the gap between the expected outcomes desired by the user to that of the systems analysts and programming team

5.3.1 Types of Testing

A number of different System Testing is there which are mentioned below:

5.3.1.1 Recovery Testing:

This type of testing is performed to tests the potentiality of the system in recovering from errors. The overall system must not fail because of errors or other processing faults. The time taken by the system to recover from failures should be within a stipulated time and tolerance limit. To check how the system responds to particular exceptions, system failures are forced by introducing certain exceptions.

5. 3.1.2 Security Testing:

System developed for processing sensitive data are vulnerable to high security risks. For various reasons people often attempt to access unauthorized data. Threats could be from external as well as from internal factors. One of the most common problems faced by system is hacking of passwords. There are many software applications that are available in the market for generating random passwords which help individuals to gain access to the system. Security testing is performed to take care of these aspects of the system security.

5. 3.1.3 Stress Testing:

To test how the system exhibits in abnormal situations, Stress test is performed. The objective of this type of test is to find the maximum limit for volume or frequency of input data, after which the system will fail. In other words to attempts to find the load that the system will be able to take during peak hours. Test cases for stress testing are designed in such a manner that it requires maximum memory and some other resources; more than what normal situation demands.

5. 3.1.4 Performance Testing:
Specifically for real time and embedded systems, performance testing is very important. It is conducted to find out the response time for a query or taking backup files or the time required for sending or receiving message. The observed response time is then checked with the expected maximum response time .The test cases for this type of testing include large files that the system will actually store during its execution in real time. It is sometimes jointly performed with stress testing.

5. 3.1.5 Regression Testing:
This system testing is conducted to ensure that recent changes in the code has not introduced new errors or adversely affected the system. To conduct regression test the already designed test cases are re-executed to confirm that the existing functionalities are working fine. It makes sure that the old code works fine even after the latest changes are made.

5. 3.1.6 Usability and Documentation Testing:
This testing is conducted to assess the the software's usability and user friendliness. Very often, it is observed that on-line help screen is provided in systems to help the user. This test checks whether proper care has been taken in documenting the development stages of the project. If the system lacks in its user friendliness aspect, then it might lead to problem during implementation as well as maintenance of the system.

5.3.2 Activities undertaken during System Testing

A number of activities are conducted as part of the system testing process. The throws light on the important activities in system testing:

5.3.2 .1 Preparation of Test Plan:
System testing starts with the preparation of the Test Plan which is a document which outlines the different facets of the system that needs to be tested. A reasonable Test Plan, with respect to the design specification, is prepared that includes the following point –

- Output expected from the system.
- Basis for assessing the output.
- The nature and the volume of test data.
- The procedure on how to use the test data.

5.3.2 .2 Specifications for User Acceptance Test: The end –user is involved in the preparation of test cases, which can be obtained from the test plan. Some of the other parameters specified are a test schedule, the test duration and the person entrusted with the responsibility of carrying out the following test - the user acceptance test.

5.3.2 .3 Designing of Test data: Test data are frequently generated by the programmers for testing the programs. The test data can be classified into two types-live and artificial data. The data extracted from the organization's files are the live test data. On the other hand artificial test data are data created by programmers to test all combination of values and formats. Artificial test data must be created in such a way that it can a truly represent the live

data which will be used after installation, by the end users. Care must be taken while selecting the nature and the volume of data.

While conducting tests on the system though sufficient care is taken to adhere to the specification mentioned in the user acceptance test document, but there still remains confusion on how the end product will be used by the user. In cases where there is a single customer (an application designed for only a particular use for a specific customer), then a series of tests are carried out to validate all the user requirements. But the case is different for the system built with a view to be used by many customers (for instance a general purpose application like spreadsheet). For this kind of software two different types of testing techniques are used called the Alpha and Beta testing.

a) Alpha Testing:

Alpha testing is conducted by the user at the developers' site. The software is used by the customer who records the bugs or errors and usage problems encountered if any. This testing is always conducted in a controlled environment.

b) Beta Testing:

This testing is conducted at one or more of the customer sites, by the end users. It is a live testing of the software. The software is tested by the customer with their own data and subsequently records and reports any errors or problems faced to the developer at regular intervals. This testing is not done by the developer.

Stop to Consider
Software developed for a larger group of use are tested using the alpha and beta testing methods

For the purpose of system testing, many organizations engage personnel's, specially trained in system testing. The bugs or errors discovered in alpha and beta testing are corrected and then after correction only the software product is installed in user's or customer's premises. The testing of complex software can consume huge amount of time and also frustrate the personnel involved in testing. The objective with which system testing is carried out is to discover every possible error or bugs that may crop up at the end-user end.

Check Your Progress
1. The first step in system testing is to prepare a document called ……
2. ……..……. is designed to test the system as to how the system behaves in abnormal situations.
3. In ……..…… live data is used in the customer's real working environment

5.4 Software Quality Assurance (SQA)

Software quality assurance is a sequence of activities undertaken all round the software engineering process. It comprises the entire gamut of software engineering life cycle.

Software quality assurance aims at producing high quality software by applying a number of procedures and standards that ensures that all the customer's requirements are met.

The purpose of SQA or Software Quality Assurance gives the management a clear and proper visibility of the software project and its products being developed. It involves auditing and evaluating the product all along the software development life cycle to validate that all the explicit and implicit requirements, standards and procedures have been followed. Adherence to agreed-upon procedures and standards is evaluated by monitoring process, evaluating products, and auditing.

Stop to Consider
The process of Software Quality Assurance involves auditing and evaluating the software product throughout the entire development life cycle

It is a set of systematically planned and coordinated actions essential for providing reasonable confidence that the software product complies with accepted technical requirements. In other words Software Quality Assurance is a collection of activities designed for evaluating the process of software development and/or maintenance.

5.4.1Objectives of Software Quality Assurance

The objectives for conducting quality assurance can be mentioned below −

- To keep track of the process of software development and the final product developed.
- To ensure that the software project is adhering to the standards and procedures as set by management.
- To inform specific individuals and groups about the activities of SQA and its results.
- To ensure that unsolved issues within the software are forwarded to the upper management.
- To recognize deficiencies if any in the product or process or in the standards, and correct them.

5.4.2 The process of Software Quality Assurance

1. Describes the requirements for software controlled system fault or failure detection, isolation, and recovery

2. scrutinizes the software products and development processes for error prevention and/ or controlled change to reduce functionality states

3. Describes the process for investigating and measuring defects and factors of reliability and maintainability

Various set of individuals like the project managers, software engineers, Software Quality Assurance people, and customers are usually involved in the activities related to software quality assurance with the role of each group being different. The Software engineers ensure

that for developing the software, suitable methods are applied and participate not only in testing the software product but also in formal technical reviews. SQA group on the other hand help the software engineer to built high quality product.

5.4.3 Components of Software Quality Assurance

SQA has six different of components which are mentioned below;

5.4.3.1. Pre-project Plan

It is prepared to ensure that the schedule, resources required for project, and budget for the project are clearly defined. A plan for development and quality assurance is also determined. The components of this pre-plan are a development plan, Resources required including both hardware and human, schedules, risk evaluation, project methodology and quality plan.

5.4.3.2. Project life-cycle component

There are two phases of a project lifecycle:

5.4.3.2.1. Development Phase:

In this phase, SQA assists to identify design and also programming errors. It is achieved through Reviews, taking Expert Opinions, and conducting Software Testing.

5.4.3.2.2. Operation Maintenance Phase:

In addition to Reviews, Expert Opinions, and Software Testing, this phase, include the specialized components which aim at improving the maintenance tasks.

5.4.3.3. Error prevention infrastructure and improvement components

This component aims at software's faults prevention and minimization of error rate .Its sub-components are Templates and Checklists, Procedures and instructions for work, Preventive and Corrective measures, Staff Training and Certification, Documentation Control and Configuration Management

- Documentation Control

5.4.3.4. SQM or Software Quality Management Components

This component controls the development and later maintenance activities. Through these components the management gains control of the software development projects. It checks the project from going behind schedule and over budget. It includes Software Quality Metrics, Project Progress Control, and Software Quality Costs.

5.4.3.5. Software Quality Assurance assessment, Standardization and Certification, components:

This component implements managerial and professional standards for the organization. It helps in the coordination among the various Organizational Quality Systems. I also establish standards for the progress of project. Some of the Components are Project process standard and Quality management standards

5.4.3.6. The human resource for Software Quality Assurance

This component initiates and supports the implementation of various components of Software Quality Assurance; recognize any deviations from the recommended improvements or from predefined SQA procedures and methods. The SQA team consists of the SQA committee, test managers, SQA unit, testers, and SQA forum members.

Check Your Progress
1. What is quality assurance?
2. What are the components of quality assurance?

5.5 Hardware/Software Selection

One of the major elements for building systems is the selection of appropriate software and Hardware. At first the kind of hardware and peripherals required for the system needs to be determined. Software compatible with the selected hardware then also needs to be being selected. Various options regarding both hardware and software available in the market are explored by the system analyst and then suitable ones are selected. The process of selecting hardware/software starts with the requirements analysis which is followed by a request for proposal from the vendors. Then the vendor evaluation process starts. After the systems are finalized it calls for contract and price negotiations, vendor selection, maintenance agreements, acceptance norms and similar issues**.**

5.5.1 Hardware Selection criteria

Selecting hardware in today's world is a significant and time consuming task. But sadly hardwires are still selected based on the reputation of the vendor only and some other factors. Few of the most important factors on which hardware should be selected are mentioned below

5.5.1.1 Determination of size and capacity

With wide variety of computers available in the market, in terms of size and capacity, it becomes difficult to select a system. The first criterion for selecting specific equipment is the capacity and size requirements. One computer system may prove be appropriate for a particular workload while it may be inappropriate for another. The deterring factor for it is the systems capacity. Important features to consider in system capacity include the following:

1. Cycle speed of system for processing
2. Internal memory size
3. Display and communication components' characteristics.
4. Number and type of attachable auxiliary storage units.
5. Availability of Systems support and utility software

Hardware requirements such as size of internal memory, disk capacity, communication ports, and the facility for using magnetic tapes, are often determined by software needs. Vendors are reliable source regarding configuration requirements who can provide appropriate information regarding minimum configuration required for using the using the software properly

File storage capacity and processing need dictate the size of auxiliary storage. In order to calculate the amount of disk space required for the system, the analyst must take into consideration the space needed for each master file, space needed for software's that includes systems software, and also the means by which backup will be taken. File size along with backup considerations; guide the decision on the number of disk drives that will be needed. Configuration should keep room for backup copies of all disks.

5.5.1.2 Computer evaluation and measurement

On the basis of performance on some standard data, computers can be compared. Use of benchmark data, generated by synthetic programs, is found to be more effective than comparing **5.5.1.2.1 Benchmarking**

Benchmarking is the method of emulating the actual processing work performed by a computer system, with the application of synthetic programs. To represent the workload of

the of the end user Benchmark programs accepts a mixture of jobs. They are capable of demonstrating techniques used by data storage equipment and can help in testing system's specific functions. With the help of this technique, the shortcomings in the equipment become evident early in the process of acquisition. Certain user organizations might insist on attaching the results to the sales contract, stating the number of transactions that can be processed within a given time period, the response time for a query and so forth.

Stop to Consider
A benchmark is the application of synthetic programs to emulate the actual processing work handled by a computer system

5.5.1.2.2 Design of Synthetic Programs

A synthetic program is a program designed to utilize a computer's resources in a manner that emulates the expected jobs and also determines the results. The artificial job stream is adjusted and re-executed to determine its impact for that specific task... This process is repeated for a number of times in order to check which tasks are better handled by a computer from a comparison set and which tasks they do not handle well. Synthetic programs often simulate different types of software and hardware features that are listed in the table below.

HARDWARE	SOFTWARE
CPU processing speed.	Scheduling algorithm
Interrupt handling abilities	Code efficiency
Memory access speed	Compilation algorithm
Printer speeds.	Virtual storage management algorithm
Peripheral channel speed.	Interrupt handling
Rotational delay for magnetic disk	File handling efficiency
Seek time for magnetic disk.	Indexing methods
Communication speeds	Communication processing procedure
	Multiple buffer handling

Table 5.1 Benchmark representatives for hardware and software.

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5.5.1.2.3 Plug – Compatible Equipment

Due to cost factors, analysts often consider using equipments for the computers not manufactured by the same computer vendor. Such type of components is referred to as plug-compatible equipment. Certain companies are specialized in manufacturing specific systems components, such as disk drives, printers, or memory units. Specific equipments from these

companies can be connected to the vendor's system in lieu of the same equipment built by the vendor. The working of the central processing unit will not be hindered even if the equipment is not from the same vendor. Plug- compatible are used because of its benefits of being low in cost in comparison with the one produced by major vendors.

Although a large number of of plug-compatible equipment from different vendors are available in the market, the analyst must carefully choose the equipment that will meet the necessary quality levels, will perform at per with the original equipment (or even better than). Moreover it must be ensured that because of the plug-compatible equipment the computer vendor does not disallow warranties and discontinue service agreements on the rest of the system.

Stop to Consider

Plug-compatible have the advantage of being low in cost as they are produced by firms specialized in producing specific components

5.5.1.2.4 Financial Factors

The payment for the acquisition of a computer system is generally handled through one of the three methods: lease, rental, or purchase. Determining which among the three options is appropriate depends on the plans and characteristics of the organization at the time of acquisition.

5.5.1.2.4.1 Rental

Computer rental is meant for short term use of a system, ranging from 1 to 12 months. Against the use of the system each month a payment has to be made to the supplier. The user as well as the supplier has the right to cancel the rental with advance notice. Because of its short-term nature, the renter enjoys great deal of flexibility in delaying its decision to purchase a system until either the availability of adequate financing or until the availability of a new generation of equipment, or till time the organization wishes. Compared to others rental is the most expensive acquisition model.

5.5.1.2.4.2 Lease

Lease is a commitment for using a system for a particular time period (generally from three to seven years). Payments, which are predetermined, remain the same throughout the duration of the lease. Payments are made annually, semiannually, quarterly, or monthly depending on the terms of the lease and it includes the cost of maintenance and equipment service. Leasing is less expensive as compared to rental. Because of longer commitment, supplier usually provides better service. Leasing provides protection against technical obsolescence, which is always a concern while purchasing computer equipment. Leasing offers definite tax advantages. Also, no capital investment is needed to lease out a computer system.

5.5.1.2.4.3 Purchase

Purchasing computers is the most common practice of acquiring computers. It has gained popularity owing to rise in lease cost. Over time, this option often costs the least, especially because of the tax advantages that can be gained sometimes.

Purchase enables the organization to take ownership of the equipment. And, in a sense the organization is locked in to the system it purchases, since changing to a different computer

system is more difficult; either the system must be sold or arrangements must be negotiated to trade it in on a different computer.

Maintenance services for both parts and labor must be acquired by the organization itself from the manufacturer, and must pay the charges, that might change every year. The charges still might be lower than with leasing or renting, depending on the terms fixed by the purchaser.

The purchase option indicates the use of depreciation to reduce taxes. In a way, depreciation which leads to deductions on income tax reduces the computer's cost to the organization.

Check Your Progress

1. What are the different financial factors affecting hardware selection

5.5.1.3. Maintenance and Support

One of the factors affecting the selection of hardware is the maintenance support for the system after its installation. Maintenance source, terms, and response times are some of the primary concerns.

a. Maintenance Source

Delivered and installed system comes with a brief warranty period during which the maintenance of the system is provided by the sales unit. After its expiry, the purchaser can acquire maintenance from numerous options but the most obvious source being the firm from where the system was purchased. Maintenance service can also be availed from companies called the third party maintenance firms which are specialized in this type of services.

b.Terms

Once the maintenance source is fixed maintenance agreement should be formulated. Besides the cost of maintenance the terms of the agreement are also important. The contract might be formulated to cover both parts and labor or to cover only labor or to cover labor along with an allowance for some parts. The terms of the contract depends on the willingness of the organization to expend.

b. Service and Response

Be available when needed makes maintenance support very useful .Two factors affecting maintenance are the response time for a requested service and the hours of support**.** In case of an emergency maintenance, the most important question is how long will it take to send an engineer or a technician for attending service? Organizations often mention in the contract the response time for a service call which some restricts to 2 hours, for some it may be day response .These term negotiations depends on the degree of dependency the user organization has on the computer system.

Check Your Progress

1. How does maintenance affect the selection of hardware?

5.5.2 Vendor Selection

In this step the vendor among the list of vendors available is selected for procuring hardware. The vendor with the best combination of reputation, service record, reliability, training delivery time, lease / finance terms & conversion schedule is selected. Some of the selected vendors are asked to give presentation of their system. The system chosen goes before through contract negotiations implementation.

5.5.3 Software Selection One of the important aspects of system development is Software selection. The search typically begins with the software, followed by hardware. For acquiring software there are two means: custom – made or "off – the –shelf" packages. It has been observed the organization prefer is purchasing packages rather than built it; it costs less than to develop the same in house. Besides, there are a few other advantages-

1. it takes much less time for a good package to make the system running than the home-grown" packages which might take weeks or months.
2. MIS personnel are freed which make them available for some other projects.
3. As the packages are generally found to perform according to stated documentation hence they are reliable.
4. Degree of risk is minimum for large – scale systems and also for programming efforts.
5. It is not easy to predict the cost incurred for "home-grown" software.
6. The user has the advantage of examining the performance of the package before purchasing it.

However, software packages are not free from drawbacks-

1. The package may fail to meet user requirements adequately.
2. First - time software users fail to clearly define their expectations from the package.
3. Modifying the package extensively in order to meet users needs usually leads to loss of vendor's support.
4. The methodology used evaluating package and its selection is often found to be poorly defined. These results in haphazard review formed on questionable selection criteria or a faulty process. It has been that price alone cannot determine quality of a software package .A systematic review of the package is important.

5.5.3.1 Criteria for Software Selection There are a number of factors that affects the selection of software. The selection can be described as below-

a) Reliability.

Reliability is the likelihood that the software executes without failure for a specified time period. It is associated with the ease of recovery and the ability to output consistent results. The questions that should be considered are as follows:

1. Can the master file be expanded?
2. Is there any error, in case a user makes, that can bring down the system?

3. What are the recovery capabilities?
4. When a part fails how much of the system will be affected?

b) Functionality

Functionality gives the description of the facilities and performance that user expects from the finished product. The following are some of the key questions to consider:

1. Are the essential elements included in the transaction file and the data?
2. Are the computations and processing performed as specified?

c) Capacity

It refers to the software package's capability to handle user's requirements for size of files, volume of transactions and reports, number of data elements etc.

d) Flexibility

It defines the ease of editing an operational program.

e) Usability

This criterion defines the effort required in order to operate the system which includes preparing the input as well as interpreting the output of a program.

f) Security-

Security is a measure of the probability that user of the system can intentionally or accidentally destroy or access unauthorized data. An important question is how well the access of software's or data file is controlled?

g) Performance.

This criterion puts emphasis on throughput, or a package performance under peak loads. Each package should be tested with user data.

h) Serviceability.

This criterion highlights on documentation and support from vendor. Complete documentation for software enhancement is important. It includes a description of the system, including system logic and instructions. Vendor support ensures adequate technical support to the user for installation of the software, its enhancements, and maintenance, especially during the first few weeks of installation.

i) Ownership

The question remains as to who owns the software after it is purchased by the user. Most of The companies lease the software for a definite time to the user. The source code is not Accessible by the user for modification as the user does not own the software.

j) Minimal costs.

One of the major factors in deciding between in – house and vendor software is the cost. Some of the costs that must be considered are development and conversion costs, Delivery schedule, Cost and frequency of software modifications and the usable life span of the package.

Stop to Consider
Reliability,Functionality,usability,capacity,flexibility,security,performance,serviceability,ownership,minimal cost are the criterion for choosing a software

5.6 System Security

The term system security refers to the protection mechanism provided to save the system from unauthorized access, theft and modifications, and unintentional or accidental damage. Security for computerized systems, involves protecting data, software, and hardware.

Security of systems is an important concern for IT systems. Although technology has provided a lot of possibility to safeguard the systems of the organization, yet there have been instances of security lapses and breaches for which the organization had to pay heavily. System administrator have put in lot of effort to safeguard the data and computing infrastructure of the organization. Organization like banks, airlines and railway are heavily dependent on IT systems and its unavailability even for a few hours can disastrous effect. Security issues cannot be underestimated at any cost because organizations business operations can get affected as a result.

Stop to consider
The degree of security can be defined by the following formula Degree of security = 1 - (No. of security failures / No. of attempt to breach security)

Security threats can be posed by both natural reasons such as Earth Quakes, Cyclones etc. and man-made reasons like riots, unrest, sabotage etc. Immediate reactive measures should be taken to deal with an attack. Moreover, one should find out the people or reasons behind each attack which can be done by means of transaction logs etc. This kind of attacks is possible due to several loopholes in the software system such as improper security protocols 'implementation etc. Such points are exploited by attackers. The entire situation surrounding attacks is depicted in Fig. 5.1.

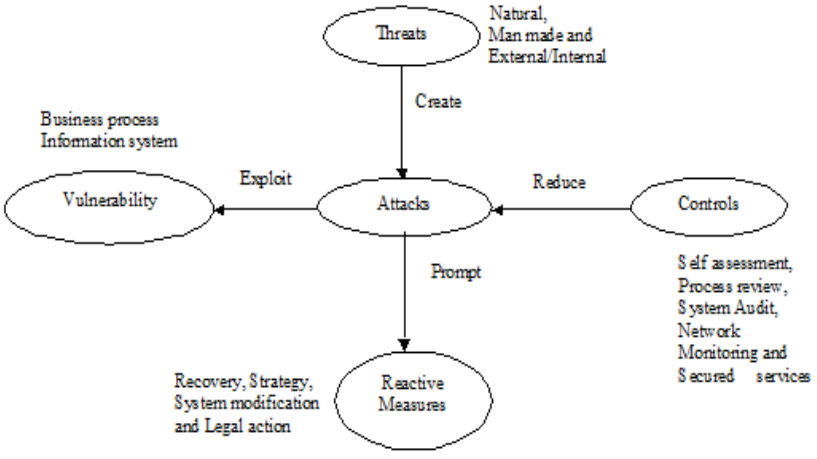

Fig 5.1 Architecture of Information Security

5.6.1 Assessment of Threats and Risks

The security of a system should be designed in proportion with the risk involved. The analysis of threat and risk includes identification of threats that are applicable to IS infrastructure, vulnerability recognition and likely loss calculation. Hence it includes identification of the source of threat that may either be external or internal. Historically virus was the only potential external threat... An organization's employee can pose serious internal threats as they know the system well and its vulnerabilities that can be targeted.

Risk Analysis

While evaluating the risks some common questions that are asked are given below.

- Are the risks due to natural reasons and their effects on the information system been made clear?
- Has it been analyzed that after a loss, whether the organization will in halt or of its like?

- Is the time acceptable for recovery of operation and has the priority of recovery been determined?

An organization's holistic outlook to security issues is underlined by its security policy. A security policy should attend to the following issues:

Authentication: To verify if the user is a bonafide one to avail the resources
Authorization: What are the privileges of the user?
Information integrity: Is it possible for the end user to modify the information?
Detection: How is the problem managed, once the same is identified?

Risk Assessment and Management

A proactive risk assessment constitutes the first step in the establishment of a sound security system. It is a continuous process that evaluates threats and vulnerabilities. It formulates an appropriate risk management policy to reduce potential monetary losses and harm to an organization's reputation. Threats can harm an institution, while vulnerabilities which are weaknesses can be exploited.

Organizations follow different approaches to analyze risks. However, all these boil down in the end, to two types of approaches, namely, quantitative and qualitative.

Potential threats

Some of the potential threats to a system are-

• Denial of service or DoS- It is an action that stops a system from operating smoothly. It may include unauthorized modification, destruction or delay of service. It usually occurs where the number of requests exceeds the maximum number of possible connections. In such cases, legitimate users need to wait for much longer time, to get a response to their requests.

• Internet Protocol (IP) spoofing- It allows an intruder to impersonate an IP address of a local system to gain unauthorized access to the system. Here, the system is made to misinterpret the incoming connection as a connection originating from any of the trusted hosts.

• A Trojan horse program – It usually performs destructive functions such as destroying data or collecting invalid/falsifying data. Such programs can be attached to e-mails.

• Viruses- These are programs that can be attached with other program, which can self-replicate. Once active, they lead to nondestructive/destructive/invalid outcomes in the

computer. The virus program may also creep into different platforms, devices or data files on a system. It may also spread through emails or through a network into other systems.

Stop to consider
DoS, IP spoofing, Trojan horse and viruses are some of the potential threats. The first virus written was a boot sector virus

5.7 Summing Up:

System implementation involves coding, testing and installation of the system and user training. The system design is translated to computer programs and required database structures are created. Individual programs are tested initially and then after merging the individual programs integration testing is done. The software in its entirety is tested called the system testing .Quality assurance guarantees that the software meets adequate technical standards.

One major element in building soft wares is selecting hardware & software that are compatible to the system. The analyst must select the suitable suitable hardware and software after a series of comparison on a numbered of factors like performance,usability,maintanability etc.The system developed should have adequate security features to fight against unauthorized access, theft and modification etc.

5.8 References and suggested readings

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5.9 Model Questions

a) Choose the correct option

1. Which requirements is the foundation from which quality is measured?

a) Hardware

b) Software

c) Programmers

d) None of the mentioned

2. Who identifies, documents, and verifies that corrections have been made to the software?
   a) Project manager
   b) Project team
   c) SQA group
   d) All of the mentioned

3. The primary objective of formal technical reviews is to find _____ during the process so that they do not become defects after release of the software.
   a) Errors
   b) Equivalent faults
   c) Failure cause
   d) None of the mentioned

4. What are the various Testing Levels?
   a) Unit Testing
   b) System Testing
   c) Integration Testing
   d) All of the mentioned

5. Alpha testing is done at
   a) Developer's end
   b) User's end
   c) Developer's & User's end
   d) None of the mentioned

B) Descriptive Question:

1. What is benchmarking.
2. Explain the different types of system testing.
3. Discuss the various methods of acquiring hardware.
4. What is system security? Explain
5. Both hardware and software are equally important in selection. Do you agree?

5.10 Answers

(A)

1) b.
2) c.
3) a.
4) d.
5) a.

BLOCK-V : Unit-I
E-Commerce

Unit Structure:

1.1 Introduction:

E-Commerce or Electronics Commerce refers to the method in modern day business, using digital media, which aids to the need of various business organizations, vendors and customers to reduce cost and enhance the quality of services and goods with increase in the speed of delivery thus contributing to effectiveness of good service.

1.2 Objectives:

The unit is an attempt to understand the concept of ecommerce, its characteristics and relevance in modern day's business ecosystem. After going through this unit you shall be able to:

- ✓ Understand the meaning of ecommerce
- ✓ Know the timeline and evolution of ecommerce
- ✓ Characteristics of ecommerce
- ✓ Meaning and concept of ecommerce
- ✓ Scope and promise that ecommerce has to offer.

1.3 Meaning and concept

In simple layman term, E-Commerce means as any form of business related transaction which is conducted online. Common example of ecommerce is buying and selling of products or any goods through the medium of internet. In broader sense, however E-Commerce can also entail various activities conducted such as online ticketing services, internet banking, payment gateways, etc. An ecommerce system can be understood by going through its phase of evolution and also the characteristics.

STOP TO CONSIDER

The section 1.3 has two sub sections 1.3.1 and 1.3.2 where the timeline and evolution of E-Commerce has been discussed giving the readers an idea about the development that has taken place in E-Commerce area and also the characteristics that any E-Commerce platform must have.

1.3.1 Timeline and evolution of E-Commerce:

In 1965, Dr. John R. Goltz and Jeffrey Wilkins, established the first E-Commerce company called Compuserve using a dial up connection.

In 1979, Michael Aldrich also known as founder of E-Commerce invented a system of shopping electronically via connection between transaction processing computer and an enhanced TV using telephone.

In the year 1982, first E-Commerce platform was launched by Boston Computer Exchange.

In the year 1992, one of the popular and early online shopping book sites with the name Book Stacks Unlimited by Charles M Stack was created.

In the year 1994, a web browser tool called Netscape Navigator was discovered by Marc Andreessen and Jim Clark which was used in Windows platform.

In the year 1995, big giants in ecommerce such as eBay and Amazon were launched.

In the year 1998, PayPal started its maiden ecommerce platform such as money transfer could be done.

In the year 1999, Alibaba came up with online shopping system and later went on to becoming an E-Commerce giant.

In the year 2000, Google started its first online advertising method with name called Google AdWords.

Period from 2005 to 2009

The E-Commerce development saw significant changes in the mentioned four years:

- ✓ In the year 2005, Amazon launched the Amazon Prime Membership which aided consumers to get free two day annual fee with a yearly subscription.
- ✓ In the same year a platform called Etsy was started to help medium and small scale retailers to sell products online. Further in 2005 an app based service called Square, Inc was launched.

Later, from the period of 2011 to 2017 there was huge expansion in the field of E-Commerce such as Google launched its online wallet payment application, Facebook started sponsored

posts and stories for ads, Apple came up with Apple Pay which is an online payment app, Instagram started with shoppable bags which made sellers to sell directly through the various social handle mediums.

Changes that have taken place in the E-Commerce industry over the years can be expressed in following points:

- Retailers have moved their business to online mode
- Local sellers have been benefitted from social media platforms
- Operational cost have significantly reduced
- Parcel delivery cost has shown a peak rise
- Artificial Intelligence and automation tools have shown utility for logistics support.
- Social media channels such as Facebook and Instagram has become a very powerful tool to reach specific consumers and increase brand awareness and sales.
- Covid-19 pandemic has effected decisions of purchasal and huge bank of consumers have moved to ecommerce platforms.

In a way, we can also say that, E-Commerce refers to the paperless exchange of business information using the ecommerce systems mentioned below.

- Electronic Data Exchange (EDI): Electronic Data Exchange or EDI is the process of electronic sharing of business information removing the need of paper mode sharing. Invoices and purchase orders can be regarded as examples of EDI.
- Electronic Mail (e-mail): Electronic mail or in short e-mail is referred as transmission and recipient of messages by digital systems in a network. An email network permits end users to receive data of various forms such as text, sounds, animated images and graphics. Users have mail boxes where they can store and receive data sent by their correspondences.
- Electronic Bulletin Boards: Electronic bulletin boards are online based communication platforms where one can request, share or discuss information on any theme. As electronic bulletin boards are accessible publicly hence they serve a source of enormous amount of information. However, electronic bulletin boards are mostly used for educational and recreational purpose; however it can also cater to other dimensions of utility such as discussions of trends related to businesses, recruit manpower, run ads, request for technical support and many more, which may be useful for small scale businesses.
- Electronic Fund Transfer (EFT): Electronic Fund Transfer or EFT is a popular method used in financial transaction done by usage of computer networks or phones. The transfer includes use of ATM's, debit card, point of sale (POS) transactions, transfers initiated by phone, automated clearinghouse (ACH) systems. This is said to be efficient as it replaces paper based transfer and takes minimum effort and time. This process can be carried out using Debit cards, Automated Teller Machines (ATM), automatic withdrawals from bank account

Check Your Progress

Question 1. Define E-Commerce with an example.

Question 2. Write a note on the evolution of E-Commerce.

Question 3. Explain briefly the changes that have taken place in the sector of E-Commerce.

Question 4. Mention few of the E-Commerce systems. Write few points on them.

1.3.2 Characteristics of E-Commerce:

The characteristics of ecommerce can be understood and conceptualized in the following points given below:

(i) Cashless Payment: E-Commerce supports the system of digital payments using various popular modes such as via usage of credit cards, debit cards, electronic fund transfer, transfer using UPI id and many other standard available electronic payment modes.

(ii) 24x7 service availability: E-Commerce systems follow a process of automated responses making the business available anytime and anywhere.

(iii) Advertising/Marketing: With Digital Marketing, E-Commerce has made the reach of products and services of businesses not confined within a particular geography.

(iv) Improved Sales: With advent of E-Commerce, business houses have seen big leap to the existing volume of sales as products can be ordered by anyone and at any time.

(v) Support: E-Commerce provides different means of support, pre sales and post sales, thus providing better services and engagement to its customer base.

(vi) Inventory Management: Inventory management is an automated feature in most of the E-Commerce solutions. Reports are generated at the blink of an eye making the whole product inventory management very effective.

(vii) Communication improvement: ecommerce bridges the gap for faster and reliable communication with business houses and its respective customer base.

Check Your Progress

Question 1. Mention the characteristics of E-Commerce.

1.4 Scope and promise of E-Commerce

STOP TO CONSIDER

The section focuses on the scope that E-Commerce has to offer in today's modern day world and also mentions the reasons of why E-Commerce shall be a promising sector in days to come.

With huge number of internet users, there is a huge scope of E-Commerce in today's modern day world. People are investing time in checking and buying items online due to the availability of varied kind of products. Big ecommerce giant organizations such as Amazon and Flipkart offer products in all categories to the consumers. Apart from that, E-Commerce platforms provide avenues for promotion and discounts attracting more volume of sales especially during festive season. The evolution of E-Commerce has been contributing in a good manner to most of the developing countries.

In India too, E-Commerce has brought up good business prospects and sellers or resellers can reach out to consumers through the medium of various marketplaces such as Amazon and Flipkart. The traffic in these websites is in the size of millions and numerous Indian sellers have received opportunity to generate good revenue for their businesses.

The E-Commerce domain has shown good growth over the span of time. With factors such as investments from industry, good support from government and with many entrepreneurs showing strong participation to use digital platform for selling their products, all these factors have pushed the development of ecommerce industry to bigger prospects. Significant development is shown in areas of grocery and consumer goods, electronics, apparels and beauty products.

Seeing the scenario, post pandemic era, the ecommerce holds a lot of reasons for becoming the future of business. The reasons are as follows:

1. Improved User experience: The E-Commerce industry has been working on improvisation of UX design, provide better customer services, effective merchandising, opt for good logistics support and this has been supported by many new generation technologies. The main aim is to help consumers to aid in buying process through optimal workflows.
2. More reach: Many E-Commerce companies have penetrated to smaller cities and towns and this is because of the availability of high speed internet services. This has also contributed to scaling up of income for many business houses which are in E-Commerce sector.
3. Increased Use of Digital Payments: As per the studies conducted by many business strategists, it is found that the future of ecommerce industry shall be dependent on mobile commerce and most of the sales in ecommerce medium shall take place using a mobile device. Acceptance of online payments have seen a huge leap due to their

flexibility of making payments anywhere and anytime making the need of digital payment service very pertinent.

4. Growth of multiple sectors:
   The industry is seeing very fast paced growth. However, in current scenario, apart from normal sectors such as electronics, apparel, accessories, etc other areas such as FMCG and online food delivery services have also picked up and are catering to normal day to day household requirements. With the advent of ecommerce, local traders and small business houses too have benefitted making the reach of their business more prominent and strong.

Self Asking Questions

Question 1. Do you think ecommerce can be considered to have a scope in today's modern day world? Give reasons in support of your answer.

Question 2. Can you cite few reasons that have E-Commerce to be the future of business?

1.5 Summing up the Unit

In this unit, we have discovered that ecommerce has a vast opportunity in the business area. In fact, those companies shall be successful who shall consider ecommerce seriously, allocating enough resources to its development. Companies and organizations who shall design their business model keep in sync with ecommerce shall be able to reap the maximum benefits in future.

1.6 References and suggested readings:

1. Joseph P.T., S.J., *E-Commerce: An Indian Perspective* PHI, 2019
2. Arora, Dr. Shivani, *Fundamentals of e-Commerce*, Taxmann.

1.6 Model Questions

Question 1. Define E-Commerce with examples.

Question 2. Write a brief note on the evolution of E-Commerce.

Question 3. Mention the characteristics of E-Commerce.

Question 4. List the scope and promise that an E-Commerce has to offer.

BLOCK V : Unit-II

Unit Structure:

1.1 Introduction

E-Business is a form of business or any kind of commercial trading or transaction which includes sharing of information using internet. E-Commerce, on the other hand, relies on the utility of information and communication pedagogies so as to validate the external actions and relationship that a business may be having with an individual, group or any other businesses.

E-Business is different from E-Commerce as it does not only include of selling or buying of goods or services but also conducting the entire businesses processes such as operations and productions, sales and marketing, customer service, logistics through the internal or external networks.

1.2 Objectives

The unit is an attempt to help the readers have a basic understanding about the following areas:

- ✓ Definition of E-Business
- ✓ Components which constitutes an E-Business
- ✓ A brief overview of an E-Business model
- ✓ Differences between E-Commerce and E-Business
- ✓ Lure of E-Commerce
- ✓ Different categories or mode of E-Commerce

1.3 E-Business

Electronic Business (E-Business) is the process of implementing a business using various technologies such as web, intranet, internet and extranet. It covers the aspect of selling and buying of products using commercial methods implemented via electronic medium and also

aids customer and technical support. E-business not only covers buying and selling of products or services online; but it also uses various communication technologies to optimize and streamline various business parts: right from the beginning to its implementation phase. E-business involves many business processes such as Customer Relationship Management (CRM), supply chain management, online order processing, etc.

STOP TO CONSIDER

In the sub sections below, we shall be able to understand the components which constitutes an E-Business, overview of an E-Business model, striking point of differences between E-Commerce and E-Business,

1.3.1 Components of E-business

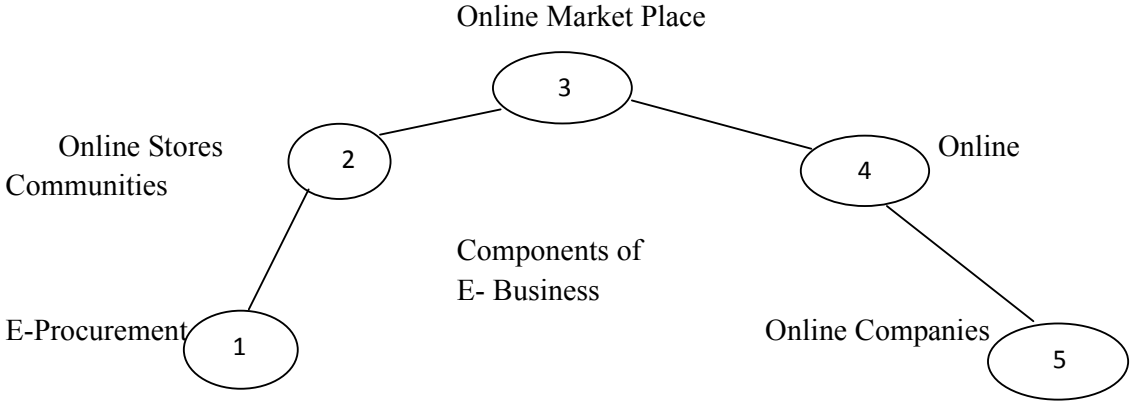

Figure 1

From figure 1, we can categorize the components of E- Business into five major categories. Let us try to enumerate briefly.

1. E-Procurement: E-Procurement is a means taken by organizations to minimize the effort and cost by sourcing services or items or products electronically. This is also sometimes known as supplier exchange where business to government, business to business, business to customer, and even sales of services is made via the medium of internet.

2. Online Stores: Online Stores also known as e-Shops, virtual store or web stores is to facilitate for buying and selling of products online. The owner of such stores must have an E-Commerce website where consumers can purchase goods using their credit or debit cards or avail cash on delivery services or even explore other payment methods.

3. Online Market Place: Online Market Place is a medium which connects prospective buyers and suppliers using internet. Such platform provides the inventory of people and transaction benefits.

4. Online Communities: Online Communities are people group formed have common goal or interest and they use internet to communicate with one another. Communication can be done between organizations and individuals to prepare transaction based decisions.

5. Online Companies: Online companies are electronic business cooperation which acts as an medium to connect singular companies and serves as online business with common transaction offer.

Check Your Progress

1. What is an E-Business?
2. Write down the components that constitute an E-Business. Explain briefly.

1.3.2 Overview on E-Business model

An E-Business model is a process which would enumerate how a company would function to provide products or services to its target groups of consumers and what shall be the roadmap in establishing the business and generate revenue. Apart from that, it shall also need to focus on how new technologies or digital market would bring value to the business. Hence, an E-Business model can encompass the following factors which are pertinent.

The factors identified are:

1. E-Business concept: The business concept is described in the form of a business prototype, goals, vision of the business, products and offers it would contain through which it could generate revenue and also have provision for scaling up. In the concept stage, a major focus shall be on market research through which the stake holders of the business shall get an understanding of the preferred choices and interests of the prospective customers. Corporate strategies are also included in implementation, business enhancement and effective decisions making.

2. Value Proposition: The term "value proposition" means the value that a business house shall offer to its customers. This can be done in the following ways:

- ✓ Reasonable Pricing.
- ✓ Better and effective service.
- ✓ On time delivery and good assistance to customers.
- ✓ Usage of technologies in the form of websites, mobile applications and digital media platforms to give customers more ways of choosing a service or a product.
- ✓ Access to varieties of products gives more scope to choose for a buyer.

3. Sources of Revenue: E-Business can have anyone or blend sources for revenue generation as mentioned below:

- ✓ Advertisement
- ✓ Affiliation
- ✓ Representative commission
- ✓ Licensing

- ✓ Sales commission
- ✓ Sponsorship
- ✓ Syndication
- ✓ Subscription

4. Required Activities, Resources and Capabilities:

To achieve the goals in a business, various activities must be performed and resources are also required. For instance, skilled employees or employees with certain capabilities can do better in specific business actions or activities.

Activities:

Certain and specific business processes are required to be implemented so as to understand and identify the cost or expenses of a activity and also outputs of each process.

Resources: Organizations need resources which can be of various types such as tangible, intangible, human and also maybe supporting resources which can aid in carrying out the processes in efficient way. Tangible resources are ones which caters to physical aspects or equipments whereas intangible resources are ones which can be patented such as intellectual property, customized software solutions and consumers data. Supporting resources may be technology utilized, communication methods used and organizational structure.

Capabilities: An organization needs skilled worker who can deliver the job to the best of his capability. E-Business is also similar to a traditional business however, with aided factors such as utility of internet, more reach of audience and also flexible way of buying any item without having to physically visit the outlet.

Check Your Progress

1. Elucidate the working of an E-Business model.
2. Highlight the factors in brief the pertinent factors which is required for an E-Business.

1.3.3 Key Differences between E-Commerce and E-Business

- ✓ E-Business is not only confined to only buying or selling of any product or a service whereas in case of E-Commerce main crux is to sell and buy services or products through the medium of internet.
- ✓ E-Commerce is a subset of E-Business.
- ✓ E-Commerce supports any kind of business transactions pertaining to money while in case of E Business it includes monetary and allied processes.
- ✓ E-Commerce utilizes only internet so as to communicate with its prospective customers while in case of E-Business, it can use extranet, internet and intranet so as they are bring able to communicate or reach the parties concerned.

Check Your Progress

1. Mention the important differences between E-Commerce and E-Business

1.4 Lure of E-Commerce:

STOP TO CONSIDER

In this section, we shall be able to understand the various important reasons that has made E-Commerce very famous in terms of performing buying and selling of goods or services through the medium of internet.

The lure of E-Commerce or attraction for E-Commerce can be understood because of the following advantages that it offers for any business.

- ✓ Lower transaction costs: if an E-Commerce site is implemented well, the web can significantly reduce order taking costs and customer service costs by automating the process.
- ✓ Scope of larger purchases per transaction using recommendation systems: Many E-Commerce sites offers a feature where if a person buys a product or an item, then one can see also what other people who ordered the book or related book bought any other product along with the book. For features like this, it prompts for a customer to buy more products online than a normal bookstore.
- ✓ Technology integration into the business cycle: A website that is well integrated into a business cycle can offer its consumers more information. For example, buying any product from an online store helps both the producer and seller to track where the product has reached using a unique consignment code and accordingly the customer can also track using the same code, making the service more reliable.
- ✓ Maintaining Larger Catalogs of products: Any organization can build a large catalog of products on the web making it easier to add or update new products information from time to time. Maintaining a paper catalog is practically not feasible.
- ✓ Improved customer interactions: With automated tools these days, it is more convenient to interact with customers in a better way with more personal engagement. For instance, if a customer receives an email when an order is confirmed, when the order is shipped and also when the order arrives. Such kind of orientation and process makes way for more purchase by a customer.

Check Your Progress

1. Write a note on the lure of E-Commerce.

1.5 Different categories of E-Commerce

STOP TO CONSIDER

In this section, we shall know the various modes of E-Commerce which shall give a basic understanding of how as per the subjects of transactions these categories have been identified.

There are basically six categories or modes of E-Commerce. They are mentioned as follows:

1. Business-to-Business (B2B): Business to Business (B2B) e-commerce covers all electronic transactions of services or goods between companies or organizations. Producers and wholesalers generally operate through this type of electronic commerce.

2. Business-to-Customer (B2C): In this type of ecommerce, electronic retail trade takes place between businesses and customers. These types of relationships are easier and dynamic. Such kind of ecommerce has developed due to the facility of web and there are many virtual stores, which sell all kind of goods such as apparels, software, books, shoes, kitchen based appliances, etc. This mode of trade also helps consumers to get more information about the product they are buying and also can get a product at a competitive price.

3. Consumer-to-Consumer (C2C): Consumer to Consumer type ecommerce covers all electronic transactions of services or goods taken place between consumers. Normally, these transactions are provided by online platforms, but are often conducted using social media platforms and websites.

4. Customer-to-Business (C2B): In a consumer to business model of ecommerce, the main working is that many skilled people make their products or services open for purchase by companies or organizations. For example, a graphics designer can make available several templates for a company logo with scope of customization by a prospective consumer and whichever one is selected, is purchased and downloaded.

5. Business-to-Administration (B2A): B2A ecommerce also known as B2G is a type of ecommerce where a business provide online services to a government. It may be through a website or a mobile application. This can be best understood by taking an example of taxes which is paid by companies or organizations to government, and this process of tax filing is executed by a third party businesses.

6. Customer to Administration (C2A): The consumer to administration ecommerce model covers all the electronic transactions taken place between individuals and public administration.

Examples of applications include:

- Education – disseminating information, distance learning, etc.
- Social Security – through the distribution of information, making payments, etc.
- Taxes – filing tax returns, payments, etc.
- Health – appointments, information about illnesses, and payment of health services, etc.

Check Your Progress

1. Discuss the different categories or mode of E-Commerce

1.6 Conclusion

In this unit, we have seen the various aspects of E-Commerce and E-Business and also how do they differ from one another. Further, with features of E-Business such as lower transaction costs, prospects for more volume of purchases using new state of art technologies, integration of technologies in to the business thus enhancing effective services and value to the business, E-Business has become a very important aspect for any business house or an entrepreneur.

1.7 References and Suggested Readings

1. Joseph P.T., S.J., *E-Commerce: An Indian Perspective* PHI, 2019
2. Arora, Dr. Shivani, *Fundamentals of e-Commerce*, Taxmann.

1.8 Model questions

1. What is an E-Business?

2. Explain the components of E-Business.

3. Write a note on E-Business model. Mention the factors that is pertinent for an E-Business model.

4. Highlight the key differences between E-Commerce and E-Business.

5. Write a note on the lure of E-Commerce.

6. Explain the different categories of E-Commerce with suitable examples.

Block V : Unit-IV

Unit Structure:

4.1 Introduction:

We already have got to know the fact that data needs to travel though a networked system in order to reach its destination. That route has to be a secured one so that nobody can intrude into and hack the data. Further implication might be losing the consistency or accuracy of data. So, to maintain accuracy, data have to be kept protected by applying some techniques. One of such techniques might be cryptography. It is associated with the process of making information secured by using some codes. Application of cryptographic tools also plays very important role in the field of E-commerce. Data are very sensitive here and need to make protected from unauthorized access. Therefore, data are converted into some other form using some codes so that they become hard to break. In other words, the actual message is encrypted by replacing the letters with some other set of characters.

In this unit, you will learn various basic and advanced concepts of cryptography and cryptography algorithms. You will learn how data are made secured and confidentiality is

maintained. We will also make a detailed discussion on various internal details related to cryptography.

4.2 Unit Objectives

After going through this unit, you will be able to:

- Understand the fundamental concepts of cryptography.
- Know various cryptographic algorithms and their works.
- Describe web security and its fundamental concepts.
- Understand the significance and functionality of Secure Socket Layer (SSL)
- Understand the significance and functionality of Secure Electronic Transaction (SET)
- Describe biometrics and its features
- Know the future of e-commerce with AI enabled tools

4.3 Cryptographic algorithms

It is already mentioned earlier that for safe propagation between sender and receiver, data need to be protected and authenticity must be protected. This is the field of cryptography which plays a very important in maintaining the integrity of data. Therefore, let us go through some important aspects of cryptography first.

4.3.1 What is Cryptography?

As already defined, cryptography actually refers to various tools providing different security services. The word cryptography has come from the Greek words which mean "secret writing". Since, data need to be secretly shared; initially the defense, military services or the diplomats had contributed towards the art of cryptography. During that time, computers were barely used and it was the duty of the code clerks to perform encryption of messages. The coded messages used to be poor as well as low-level. However, with the advent of computers and newer technologies, cryptographic algorithms have become more powerful and high level encryption could be performed. There are two key terms that in cryptography: encryption and decryption. Encryption is the technique through which plain text data are converted to cipher text through incorporating keys. Decryption performs the reverse by breaking the keys so as to receive the actual data in plain text form.

4.3.2 Goals of cryptography:

The following are the primary goals that cryptography always tries to achieve.

a. Integrity of data: This mainly deals with checking if any alternation of data has occurred. Data may get modified by any unauthorized access which results in loss of integrity of data. Data must be kept intact so that the receiver receives what was actually being sent by the sender.
b. The fundamental security service provided by cryptography is the preservation of confidentiality. The data privacy or security is maintained by denying unauthorized access. It can be achieved through the use of cryptographic algorithm.
c. Authenticity refers to the identification of the actual sender. It tries to confirm to the receptor that the message being passed has come from a verified originator. There are two variants of authentication: message and entity authentication.
    i. Message authentication refers to identification of the message originator.
    ii. Sometimes, data are received from entities like websites. Entity authentication refers to the identification and verification of the entity.
d. Another important security service that can be enabled in a system is the non-repudiation property. This property tries to ensure that accountability of the data originator. The originator cannot refuse that it has not created the message. Non-repudiation property comes into play when a dispute arises during transmission between the sender and the receiver.

4.3.3 Methods of cryptography:

Let us know some important terminologies of cryptography and how useful they are.

a. Plaintext: The actual message generated by the sender. It is represented in terms of set of character strings. The plaintext format is converted into encrypted format with the help of the key.
b. Keys: The plaintext messages are transformed by a function parameterized by a key. Both the sender and the intended recipient must have agreed on the key. If the unauthorized intruder does not know the key to decrypt the message, he won't be able to break it despite being able to hack the coded data. The data integrity, consistency, accuracy, authenticity and security of data depend much on the keys being inserted. Therefore, keys must be kept secret so that no other except the intended can break it.

c. Ciphertext: The encryption process results in generation of Ciphertext. Ciphertext is transmitted across the network and the recipient receives it. It is the task of the recipient to decrypt it using key in order to generate plaintext.
d. Encryption: When the plaintext is inserted with keys, it becomes coded and actual plaintext becomes Ciphertext. This process of creation of Ciphertext is termed as encryption. The key used during encryption process is known as encryption key. It is the key used in sender side to compute Ciphertext.
e. Decryption: It is the reverse process of encryption which recovers the plaintext from Ciphertext. Decryption occurs at the receiver's side using decryption keys. Decryption keys are not always identical to encryption keys. They are inserted along with the Ciphertext to compute the plaintext.
f. Cryptanalysis and Cryptanalyst: Sometimes, an intruder may get into the communication channel and copy messages from there. Then he might put his own messages into the legitimate messages or modify them, before it gets into the receiver. Cryptanalysis is the art of decrypting the ciphers. The person who breaks ciphers is known as cryptanalyst.

The encryption method is of two different categories: Substitution and transposition ciphers.

i. Substitution ciphers: Each letter of plaintext is replaced by another letter or group of letters. Like A is replaced by Q, B with V, C with F and so on. One very well-known substitution ciphers is Caesar cipher. In this method, a is replaced with D, b is replaced with E, c is replaced with F and so on. In other words it can be said that Caesar cipher replaces every character with the third character towards its right. Therefore, 3 becomes key here. The key value can be modified instead of always keeping it 3. The following figure resembles this:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

Fig1: Substitution using Caesar cipher

Now, the encrypted code of the string "admrcz" will be"DGPUFC". Order of plaintext symbols are preserved using substitution ciphers but they are disguised.

ii. Transposition ciphers: In contrast to substitution ciphers, transposition ciphers do not disguise the symbols; rather reorder the plaintext letters. A common

transposition cipher is columnar transposition where the key is represented by a word or a phrase. The key must not contain any repeated character. For example, let us consider the key as COUNTMAP. The plaintext message is "please share your profile" which needs to e encrypted. The key will number the columns. Column 1 will be the letter closest to the start of the alphabet and so on. The plaintext is written horizontally in order to fill the matrix. Finally, the Ciphertext is formed by columns starting with lowest key value. Considering the following matrix diagram, the Ciphertext formed will be as follows:

C	O	U	N	T	M	A	P
2	5	8	4	7	3	1	6
p	l	e	a	s	e	s	h
a	r	e	y	o	u	r	p
r	o	f	i	l	e		

Fig 2: Transposition cipher

The Ciphertext corresponding to the plaintext "please share your profile" would be "SRPAREUEAYILROHPSOLEEF".

To break transposition cipher, it must first be understood that this is transposition cipher. Moreover, the number of columns must be known in advance.

4.3.4 Basics of Cryptographic algorithms:

Cryptography starts with the fundamental assumption that the cryptanalyst knows the encryption and decryption methods. This infers the fact that the methods are no longer secret. Moreover, every time the method changes and new methods are adopted, secrecy of methods becomes impractical. This is where keys come into play. Keys are relatively short, exclusively secret strings that work as a parameter to the general methods of encryption. Thus, we have a principle known as Kerckhoff's principle which says:

All algorithms must be public; only the keys are secret

There are two classes of cryptographic algorithms: Symmetric key algorithms and Asymmetric key algorithms.

4.3.4.1 Symmetric key algorithms: The encryption algorithm that uses the same key for both encryption and decryption is known as Symmetric key algorithm. Cryptographic algorithms can be implemented either by hardware or software.

Let us discuss a very widely used cipher known as DES (Data Encryption Standard) for securing data. The plaintext is encrypted in 64-bits blocks resulting in 64-bits Ciphertext. The algorithm involves 19 different passes and 56-bit key. Initially, the 64-bit plaintext is converted into key independent transposition. The final stage generates exactly the inverse copy of the initial transposition. The stage before the final phase does the exchanges between 32-bit leftmost and 32-bit rightmost bits. Remaining 16 stages are functionally identical. They take two 32-bit inputs and produce two 32-bit outputs. The left output is simply an identical copy of the right input. And the right output is generated by performing the XOR of the left input and a function of the right input and the key for the stage. In each 16 iterations, different keys are used. As already mentioned, decryption key is identical with the encryption key. However, the steps are run in reverse.

An upgraded version of DES is 3DES, which was developed to overcome the drawback of DES. It applies the DES algorithm thrice so as build a strong, hard to break Ciphertext. 3DES is extremely useful in payment systems and technologies in various industries like finance. Standard cryptographic protocols such as TLS, SSH, OpenVPN use this method.

As we already know that no encryption algorithm lasts for a very long time and they continually change; DES also began to approach its end. Another new standard of technique was introduced, known as AES (Advanced Encryption Standard). The design issues were- just like DES, AES must be made public. Both software and hardware implementations must also be supported. Both encryption and decryption must agree on the same key with the supported key lengths of 128, 192 and 256 bits and a block size of 128 bits. On the basis of some important key points like security, simplicity, efficiency, flexibility and memory requirements, some standard techniques were proposed. Out of which Rijndael cryptographic standard was taken up. Rijndael supports key length and block size of 128 bits to 256 bits in 32-bits steps. Both hardware and software implementations are possible. The function has three parameters: *plaintext,* consisting of an array of 16 bytes input data, *Ciphertext,*

consisting of 16 bytes output data and a 16 bytes *key*. Initially, an array *state* is assigned the plaintext. It is copied to state in column order, with first four bytes into column 0, second four bytes into column 2 and so on. At every step of computation, the array state is modified. Both substitution and permutations are performed on *state*.

The key is expanded into same sized 11 arrays of size 4X4 each. During each round of computation, one such array will be used. The first array is XORed with the 16-bytes plaintext stored in state at the beginning pass. In the next 10 rounds, the content of *state* is transformed by four basic steps-

a. Byte-for-byte substitution of state.
b. Rotation of each of the four rows to the left.
c. Mixing up each column by performing multiplication between the old column and a constant matrix.'
d. Finally, the key and the state array are XORed.

Decryption is done by running the algorithm backward.

4.3.4.2 Asymmetric key algorithms: In contrast to symmetric algorithms, asymmetric algorithms involve multiple keys for encryption and decryption. Two distinct but otherwise related encryption keys are used here: public key and private key. That is why, asymmetric key algorithms are also known as "Public key encryption". The advantage of this algorithm is that it provides greater amount of security with public key being used for encryption and private key for decryption. Another advantage of this algorithm is the authenticity. Since, encryption and decryption are done by two different keys; this method always makes it sure that data is always received by the intended recipient.

RSA is considered to be one of the strongest algorithms that is hard to break. Its name comes from the initials of its three discoverers Rivest, Shamir and Adleman. Keys are at least of size 1024 bits. This public key algorithm is based on some number theory principles and is one of the most secured encryption techniques.
The algorithm begins by considering two relatively large prime numbers p and q and then computing the products $n= p\ X\ q$ and $z= (p-1)\ X\ (q-1)$. A number relatively prime to z is chosen and let us call it d. Finally, e is chosen such that $e\ X\ d= 1\ mod\ z$

The plaintext is divided into blocks so that each plaintext P falls within the range $0 \leq P < n$. Each block is grouped into group of k-bits, where k is the largest number which makes $2^k < n$.

Encryption of a message P requires to compute $C = P^e \pmod{n}$. Decryption of C requires to compute $P = C^d \pmod{n}$. Encryption requires the values of e and n to be known in advance. Similarly, decryption needs d and n. Encryption and decryption functions are inverses of each other. Therefore, public key consists of the pair (e, n) and private key consists of (d, n). The cryptanalyst could easily find out p, q and then z, if he could factor n. Security of RSA algorithm is based on the difficulty in factoring large numbers. It is difficult to factor large numbers.

4.4 Digital Signatures:

It is already known to us that the authenticity of any report or document can be proved with the handwritten signature. However, it becomes a bit complicated if the document needs to be transferred and authenticated through computational means. A method must be devised which will replace the handwritten signature with a different one. It is always a public-private affair.

Digital signatures are useful when a receiver wants to identify the sender. If one party sends a signed message to another, the destination party must be able to authenticate the source from which the message was being sent. Message authenticity is maintained by having the public key of the sender. Using this key, the receiver verifies that the sender has actually sent the message.

Data integrity is achieved by identifying the sender with the help of the key. If the data is modified by the attacker, digital signature verification of the sender will fail. The receiver will deny that message. When the sender sends a message using a signature, it cannot deny sending it later on. If any dispute arises between the communicating parties, digital signature will help the receiver prove the fact that the message was actually being sent by the senders itself.

4.4.1 Symmetric Key Signatures:

We might think of a situation where there is a party sending message through another one. That other system works as a central monitoring party that knows everything about the

system, like who is sending or where to forward etc. It is the authority to which each communicating party interacts.

When a user wants to send a signed plaintext message to another user, it generates a message encrypted with a key. Then the message is sent to the central authority which decrypts and sends the message in terms of plaintext format to its receiver. Apart from the plaintext, the recipient also receives a signed message.

Now, in case a rift happens between the communicating parties and the sender denies sending the messages, the signed messages play an important role here. The central system would not have accepted the message unless it was encrypted with the key. Furthermore, it would not have accepted the message unless it was signed by the central authority. This in turn proves that the communication was initiated by the sender itself, not by another party.

The underlying condition of symmetric key cryptography is that the communicating parties must have trust on the central authority.

4.4.2 Public Key Signatures:

The symmetric key signatures have a problem of allowing the central system to read all signed messages. Considering the public key encryption and decryption algorithms having an additional property $E(D(P))=P$ apart from the usual property $D(E(P))=P$. With these assumption, sender A sends signed plaintext messages to receiver B by transmitting $E_B(D_A(P))=P$; E_B being B's public key and D_A being A's private key.

When B receives the message, it decrypts using its own private key. This yields in generating $D_A(P)$. Then B applies E_A to generate the original plaintext P.

Like in the previous case, if any adverse situation occurs between A and B, B can produce both P and $D_A(P)$. A message which was encrypted with D_A can be decrypted applying a public key E_A in order to generate the plaintext P. This is how; it can be proved that A was the actual sender of the messages. However, this remains true till the time D_A is true. Once D_A is disclosed, A can no longer be proved to be the actual sender as anyone could have sent the message.

4.5 Web security:

The motto of communication network is to perform errorless transfer of data. The bits transferred must be accurately received without any modification being made in between. No unwanted attacker must be allowed to intrude so that integrity of data can be maintained. For this, security must be ensured within the network. Particularly, security breaching may occur through viruses, worms etc. They may destroy very sensitive, valuable information. Similarly, e-mail security also plays an important role in a network system.

Firewalls are the mechanisms which inspect everything that enters and leaves a network. They employ packet filtering technique having the advantage that each packet has to pass through two filters and an application gateway. No other alternative route exists. Packet filters have some extra functionality which allows the packets to transit once thorough inspection of the packets is done. One filter checks the incoming packets and the other checks the outgoing packets. And in between the application gateway exists. No packet is allowed to get in or out without having to pass through the gateway. A company may block the IP addresses with ports for application services like telnet or finger so that nobody can log in. Application gateway works at the application level apart from inspecting the raw data. An e-mail gateway can examine all incoming or outgoing messages in order to discard or transit messages.

It is worth mentioning here that not only web security, secure connections are also essential for good network architecture.

4.6 Secure Sockets Layer (SSL):

Earlier, web consisted only of static web pages. However, with the growing demand of newer transaction oriented technologies, requirement of secured connections are increasing. Today, people are more inclined towards doing financial transaction though online mode, online banking or purchasing items through credit/debit cards. Therefore demand ensuring security during online transactions has led to the introduction of a software package called Secure Sockets Layer (SSL). It builds a secure connection between the sockets of client and server. SSL works by first establishing a parameter negotiation between the communicating parties. This includes maximum message size, quality of service and other parameters. Once it is accepted, mutual authentication of both sides are carried out. This technique verifies the

identities of communicating partners to be non-malicious. Once this is also established, the actual data communication proceeds, keeping the integrity of data intact.

SSL is placed in between application layer and transport layer of the protocol stack communicating between the client and server over TCP. The main task of SSL is to handle compression and encryption. When HTTP is over SSL, it is called HTTPS (Secure HTTP). Not only is the web browsers, SSL also used in most of the common application platforms. It supports various algorithms of compression, cryptographic algorithms etc.

SSL has two sub-protocols: one for establishing secure connection and other for using it. For instance, Machine1 wants to communicate with Machine2 by sending a request message. The request message consists of Machine1's preferences of cryptographic and compression algorithms. When Machine2 receives the message, it replies back with reply messages. First message contains the choice of algorithms among the preferences set by Machine1. Next message contains a certificate containing its public key so the Machine1 can verify Machine2. Other messages may also follow like verifying Machine1's identity. Finally, a message flows to Machine1 asking it to transmit.

Machine1 responds by sending a random 384-bit premaster key encrypted with Machine2's public key. Upon receipt of the message, both Machine1 and Machine2 can start computing the session key. Machine1 informs Machine2 of establishment of sub-protocols and requests it to switch to a new cipher. Subsequent two messages are the acknowledgements Machine1 receives from Machine2. SSL supports multiple cryptographic algorithms. One of them supports triple DES encryption with three separate keys and SHA-1 for message integrity. Systems which require high end security uses this technique. RC4 encryption with 128-bit keys and MD5 message authentication are used for e-commerce application.

Let us see how the second sub-protocol works. The requests made through browsers are broken into units up to maximum 16KB. Each unit is separately compressed if needed. The secret key is derived and premaster key is attached with the compressed text and hashing algorithm is applied. The hash is appended to each unit as MAC. The compresses unit along with MAC is again encrypted and a header is attached. This combined unit is transmitted over TCP connection.

SSL provides guaranteed confidentiality with encryption; with disguising information, not to be deciphered by anyone except the recipient. Integrity of data is also preserved with

maintained authenticity. However, cost is a big issue as the whole setup incurs more expenditure. Speed is also a major drawback as various procedures need to be established back and forth prior to actual communication. Moreover, older versions of SSL are vulnerable to threats subsequently leading to insecure encryption.

4.7 Secure Electronic Transaction (SET):

Today, the importance of e-commerce is tremendous in our daily life. There is a reduced transaction cost, sales and purchase cost, operating cost and many more. With the enrichment in business, there is a parallel growth in production, revenue, customers and services; eventually leading to growth in the global market. Services play a very important role in businesses. One of such kind may be the security services. In an e-commerce site, everyday thousands of financial transactions may happen electronically. Keeping those transactions secured from unauthorized access is a huge matter of concern. Otherwise, a huge penalty has to be paid for it.

Secure Electronic Transaction (SET) is an open-source protocol ensuring security of electronic transactions made though credit or debit cards. It is theoretically one of the best cryptography based protocol used for secure payment processing. During online payments, customer's card information is fed. Keeping that information safe and confidential are important to ensure integrity of data.

SET was initially proposed and launched by a collaborative effort of VISA and MasterCard in February 1996. The SET algorithm also authenticates merchant's information so that legitimate seller can accept user's transactions. SET certifies that the merchant has a relationship with a financial institution. Similarly, authentication of cardholder also plays a very significant role to certify that a legitimate user uses the card. It tries to verify that the cardholder is a registered user of a valid card.

Customer's payment and card information travel across the network which is vulnerable to change. Confidentiality of information is provided by the DES encryption technique. The seller can never get to see the card number is an important feature of SET. Online payment includes many crucial information like order information, customer's personal information and card details to be inserted into the merchant's site. Integrity guarantees this information is not modified during transit by blocking the details and prevents hackers from stealing them.

The payment gateway certifies that there is a standard and automatic payment process. There are some standards to be followed which enable the Certificate Authority (CA) to issue certificates to all the participants.

4.8 Biometrics

Technological advancements have contributed towards accessing, analyzing, verifying and recognizing biological data. Biometric system is specifically used for the purpose of identification of individuals. Today, government and many business organizations use biometric systems to acquire more information about every individual. Another important advantage is more security features can be imposed on system by validating the consumers. Like fingerprint or face recognition systems, handwriting patterns, iris recognition, retina scanning, voice recognition can lead to recognition of human beings. These features may be considered physiological features. However, there are behavioral features which also fall under the biometrics; like for example someone's typing patterns, mouse movement, finger movement or signature etc.

In an e-commerce site, addition of biometric features can provide extra security to it. It also has the advantage of protecting customer information. On the other hand, the presence of the customer during a transaction can be assured. Authentication using biometric verification has been widely used in practices in areas like corporate sectors, public sector organizations, and security services or in point of sales. Security factor is becoming part and parcel of E-commerce as its span is expanding everyday. Need for security feature proportionately increases with business expansion.

Biometrics has the advantage of being convenient or easy to use. At the same time, data cannot be easily stolen. They are less prone to change as physiological characteristics remain mostly same throughout the life time.

However, the amount of biometric systems employed is very limited. The installation and running cost increases in accordance with the cost of hardware/software. Moreover, if the system fails to capture biometric data, it may eventually lead to false acceptance or false rejection. Biometric data are also susceptible to hack.

4.9 Future of E-commerce:

The e-commerce sector is constantly upgrading and more changes have occurred by incorporating new technological innovations. There is always a battle to among the companies to prove oneself to be the best. Systems are increasingly becoming expert and more artificial intelligent based. They can now make predictions based on consumer's previous experience. In other words, systems can be more personalized taking care of each person's preferences and choice. Parallely, there will be a huge challenge in protecting the user data as well as building such a secured system.

Social media can be one medium through which business can proceed. In fact we have seen businesses are operational via social media. In future, it is expected that social media would become a great medium for launching, promoting and improving online business.

When there is a blend of high resolution photos, videos and Augmented Reality, things become more attractive as well as grabs customer's attention. Visual commerce is thought to be the new area of e-commerce having a great significance in increasing revenue.

It is already mentioned above that efficient, intelligent and expert systems are becoming part of e-commerce business. We already have voice enabled deep learning based intelligent systems like Google Assistant, Siri or Alexa. If tools could have been developed for searching or purchasing products, ecommerce would have moved to a great level. Similarly, AI assistants could be developed to respond to queries or concerns, taking care of orders of the customers. AI enabled chatbots, virtual assistants may give new norms to online shopping.

To boost up earnings and innovation of new AI enabled techniques and tools have led to the development of a new concept called dynamic pricing. This is a technique to optimize pricing of products.

4.10 Summing Up

- Cryptography is the technique of keeping data safe and protected from unauthorized access. It enables data to remain consistent and accurate throughout the path from source to destination. Data are very sensitive and they need to be routed in coded form. Moreover, confidentiality and authenticity play very significant role in cryptography.

- Cryptographic algorithms are applied to the plain text messages at the sender's side to generate coded or encrypted forms. The encrypted form is known as Ciphertext which actually passes through the medium. At the receiver's end decryption algorithms are applied to the Ciphertext in order to generate the actual plain text back.
- During encryption, encryption keys are used. Decryption keys decrypt the coded messages.
- In substitution encryption method, each character of the string is replaced by another character. Substitution ciphers preserve the order of plaintext symbols but they are disguised. Rather transposition ciphers do not disguise the characters rather they are rearranged. The key does not contain any repeated character and data are also represented in matrix format.
- Cryptographic algorithms continually changes. Therefore, it is impractical that the methods must be kept secret. However, keys play the most significant role in maintaining secrecy and confidentiality of data.
- When the sender and the receiver agree on the same key for both encryption and decryption is known as Symmetric key algorithm. DES is one of such kind of algorithm.
- Asymmetric algorithm involves multiple keys for encryption and decryption. Two distinct but otherwise related encryption keys are used: public key and private key. It provides greater amount of security. Public key being used for encryption and private key for decryption. Asymmetric key algorithms are also known as "Public key encryption" methods. Authenticity is another advantage of this algorithm. RSA is one of such kind of algorithm.
- Digital signatures are useful when a receiver wants to identify the sender. If one party sends a signed message to another, message authenticity is achieved by having the public key of the sender. Using this key, the receiver verifies that the sender has actually sent the message. Similarly, data integrity is also achieved by identifying the sender with the help of the key.
- The communication network must perform errorless transfer of data. However, security breaching may occur through viruses, worms etc. They may destroy very sensitive, valuable information. Web security tried to ensure security within the network.

- The demand for ensuring security during online transactions has led to the introduction of a software package called Secure Sockets Layer (SSL). It builds a secure connection between the sockets of client and server.
- The main task of SSL is to handle compression and encryption. When HTTP is over SSL, it is called HTTPS (Secure HTTP). SSL provides guaranteed confidentiality with encryption; with disguising information, not to be deciphered by anyone except the recipient.
- Secure Electronic Transaction (SET) is an open-source protocol ensuring security of electronic transactions made though credit or debit cards. SET algorithm authenticates merchant's information so that legitimate seller can accept user's transactions. Confidentiality is retained by DES encryption technique.
- The future of e-commerce can be thought as a blending of AI enabled techniques and tools to business.

4.11 Key terms

- **Cryptography**: It is the technique of keeping data safe and protected from unauthorized access. While transferring data between two stations, consistency, accuracy and authenticity are preserved using cryptography.
- **Cryptographic algorithm**: Cryptographic algorithms are applied to the plain text messages at the sender's side to generate Ciphertext.
- **Encryption Keys**: These play the most important role in producing Ciphertext. Keys are applied to the algorithms to generate data in coded form.
- **Symmetric key algorithms**: Symmetric key algorithm uses the same key for both encryption and decryption.
- **Asymmetric key algorithms**: Asymmetric algorithm involves multiple keys for encryption and decryption. Two keys are used here: public key and private key.
- **Symmetric Key Signatures**: Symmetric Key Signature places a third party between the communicating stations. That hub works as a central monitoring party that knows everything about the system.
- **Secure Sockets Layer (SSL)**: Secure Sockets Layer (SSL) is a package that ensures more security during online transactions. It builds a secure connection between the sockets of client and server.

- **Secure Electronic Transaction (SET)**: It is an open-source protocol ensuring security of electronic transactions made though credit or debit cards.
- **Biometric**: Biometric features can provide extra security to an e-commerce site. It is primarily used for the purpose of identification of individuals.